REFEREE REPORT

by professor PhD Nikolai Manev,

The Institute of Mathematics and Informatics – BAS

on the dissertation thesis of **Tedis Arben Ramaj** entitle

"Algebraic methods for studying some combinatorial configurations and their applications"

Submitted for acquiring **"Ph. D. degree**"

Area of Higher Education: **4. Natural Sciences, Mathematics and Informatics**,

Professional Field: **4.5 Mathematics**

Ph.D. program: **"Algebra, topology and applications"**

Scientific Adviser: **associate professor Silvia Boumova**

associate professor Maya Stoyanova

1. General description of the procedure and the applicant

Tedis Ramaj was born in 1990 in Elbasan, Albania. She received Bachelor degree in mathematics in 2011 at the University of Elbasan and MS degree with subject mathematics in 2013 at the University of Tirana. She started her PhD study in 2018 in the PhD program "Algebra, topology and applications" (education in Rnglish) at the Faculty of Mathematics and Informatics of the Sofia University "St. Kliment Ohridski". She passed successfully this educational program and according to order РД 20-668 from 26.02.2021 has permission to defense her thesis. Since 2013 Tedis Ramaj has been full or part time assistant professor at the University of Elbasan, Polytechnic University of Tirana and the Tirana University. She has taught Algebra, Geometry, Analysis, Differential equations, etc.

I was appointed to be a member of the Scientific Jury by order № РД 38-129 / 01.03.2021 of the Rector of the Sofia University "St. Kliment Ohridski". In such a capacity I received all required documents (in digital form) that concern the procedure. The documents shows that the applicant fully meets the minimal national requirements according to the Act on Development of the Academic Staff in the Republic of Bulgaria as well as the Regulations for the conditions and rules for acquiring PhD degree of the Sofia University "St. Kliment Ohridski".

2. Approbation of the results in the dissertation

The results presented in the dissertation have been published in 3 papers, all with her scientific advisers as coauthors. One paper has impact factor. The rest papers are indexed by MathSciNet and zbMATH.

The papers have not been used for acquiring other degrees or for occupying positions.

3. Assessment of the personal contribution of the applicant in joint works

According declaration of coauthors and my discussion with them I can conclude that the personal contribution of Tedis Ramaj to joint papers is equipollent.

4. Impact of the results on the work of other scientists

Tedis Ramaj has not reported an appearance of citations of her papers but it is not required by the Act on Development of the Academic Staff in the Republic of Bulgaria. The nonexistence of citations is easy to explain – Two papers have been just published and the third one is with status accepted.

5. Quality of the dissertation Abstract

The Abstract contains 26 pages and presents correctly the content of the chapters and the spirit of the dissertation as a whole. It contains also a summary of main results obtained in the thesis.

6. Description and analysis of results in the thesis

The dissertation thesis contains 86 pages and consists of Introduction, three chapters and Bibliography with 45 titles. The main contributions of Tedis Ramaj are correctly and completely listed at the end of Introduction on page 17 of the thesis. In my opinion they are solutions of difficult and interesting problems. On pages 19 and 20 the papers and the 6 conference talks presenting the author's results are listed. The author's results are given in Chapters 2 and 3.

Here is a short description of the content of the Ramaj's thesis.

In Introduction after short historical remarks about orthogonal arrays and their applications a review of the content of the thesis, section by section, is made. The main results in each section are underlined.

Chapter 1 contains the notions and facts necessary for reading and understanding the rest of the thesis. It starts with describing basic properties of Orthogonal arrays and relation of the subject to Coding theory. Next some classes of orthogonal polynomials are considered. Special attention is paid to Krawtchouk polynomial and their properties (Section 1.4) since they play fundamental role in Coding theory and the theory of Orthogonal arrays as well as in other mathematical areas. Section 1.4 ends with an introduction to additive characters and a review of Delsart's results having fundamental value both to Orthogonal arrays and Coding theory. In Section 1.5 the Delsart's results are applied to obtain linear systems satisfying by distance distributions.

In Chapter 2 Tedis Ramaj develops further the recently known results. She obtains systems of linear equations whose solutions are potential distance distributions of an orthogonal array with respect to a point. The knowledge of potential distance distributions of an orthogonal array and its derivatives is an important step to its construction or to proving its nonexistence. However the computing of all nonnegative integer solutions of the considered linear systems is a task that complexity increases exponentially with increasing of parameters. Also the task is not suitable for parallel processing. Hence one has to optimize between execution time and required dynamical memory. Even very small theoretical reduction of complexity is essential. The thesis presents theoretical parts of Tedis Ramaj's investigations and does not demonstrate the difficulties in development of the used software (based on Maple) and its modification for any studied case. I estimate these efforts as her achievement, too.

Among the theoretical results obtained in Chapter 2 I will to underline the Theorem 2.3.1. It not only contributes to simplifying calculations but it is used in Chapter 3.

In Section 2.5 using methods and algorithms given in the previous sections of Chapter 2 the nonexistence of OA(108,17,3,3) and OA(108,16,3,3) is proved. These facts improve the known lower bound (from 4 to 5) for indexes of orthogonal arrays over Z_3, strength 3, and length 17 and 16. Also the structure of OA(1458,16,3,5) is determined.

Chapter 3 is devoted to research in area of Coding theory that is subject of intensive and long-standing investigations, namely, covering radius of codes. In this chapter orthogonal array are treat as codes over Z_q and the problem is upper bounds for their covering radii to be found. All obtained results are theoretical.

In Section 3.2 it is proved that any OA(M,n,q,t) orthogonal array C as a code has covering radius

$$\rho(C) \leq n-t.$$

With the additional condition $n-t \geq q$ the bound is

$$\rho(C) \leq n-t-1.$$

The obtained bounds are sharp in sense that there exist orthogonal arrays where equality holds. Such examples are given in the Thesis.

In Section 3.3 the above bounds are improved. It is proved that if $n>2(t+q-1)$ then it holds

$$\rho(C) \leq n-t-2.$$

7. Critical remarks

The Thesis is well readable. I have no essential critical remarks. I have noted only several typos.

CONCLUSION

The dissertation thesis of Tedis Ramaj contains theoretical results that are original contribution to the studied mathematical area. The thesis fully meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the Regulations for the conditions and rules for acquiring PhD degree of the Sofia University “St. Kliment Ohridski”. **She possesses profound knowledge and professional skill in her mathematical area and demonstrates qualities and abilities for carrying out self-depended investigations.**

Based on the aforesaid in this report **I give positive estimation of the considered dissertation and strongly recommend the Scientific Jury to confer on Tedis Ramaj the educational and scientific degree ”Doctor” in area of Higher education 4. Natural Sciences, Mathematics and Informatics**; **rofessional field: 4.5 Mathematics; Ph.D. program: “Algebra, topology and applications”**

15.04.2021 г. Signature:

Sofia Prof. N. L. Manev