REVIEW

by Prof. Dimiter Genchev Skordev
of the work of Prof. Alexandra Andreeva Soskova
"COMPUTABLE STRUCTURE THEORY :
JUMP OF STRUCTURE, CODING AND DECODING",
presented as a dissertation
for obtaining the scientific degree
"Doctor of Mathematical Sciences"

Overview

The dissertation is in English with a length of 270 pages, and the author's summary is in Bulgarian and English. The main text of the dissertation is presented in seven chapters with the following titles and lengths:

1. Introduction (15 pages)
2. Preliminaries (25 pages)
3. Jump of a structure (25 pages)
4. Strong jump inversion (33 pages)
5. Effective embeddings and interpretations (45 pages)
6. Cohesive powers (50 pages)
7. Cototality and the Skip Operator (52 pages)

The bibliography includes 162 titles, Prof. Soskova being the author of 5 of the cited works and a co-author of another 13 (all of them are on topics different from the one in her PhD dissertation, defended in 1990). Ten of these 18 papers have been declared as scientific papers included in the dissertation. They are the following:

[Sos07a] "A jump inversion theorem for the degree spectra", Lecture Notes in Computer Science, 2007.[1]

[SS07] "Jump spectra of abstract structures" (joint with Ivan Soskov), Proc. of the 6th Panhellenic Logic Symp., 2007.

[SS09a] "A jump inversion theorem for the degree spectra" (joint with Ivan Soskov), J. Logic Comput., 2009.

[SS09b] "Some applications of the jump inversion theorem" (joint with Ivan Soskov), Proc. of the 7th Panhellenic Logic Symp., 2009.

[CFH$^+$18] "Strong jump inversion" (joint with 6 other authors), J. Logic Comput., 2018.

[KAV19] "Coding in graphs and linear orderings" (joint with two other authors), J. Symb. Logic, 2019.

[ACG$^+$20] "Interpreting a field in its Heisenberg group" (joint with 8 other authors), submitted.[2]

[1] It is cited as [Sos07] in the author's summary.

[2] An additional letter by one of the authors dated December 23, 2020, attests that the work is in fact accepted in J. Symb. Logic.

[DHM+19] "Cohesive powers of linear orders" (joint with 5 other authors), Lecture Notes in Computer Science, 2019.

[DHM+20] "On cohesive powers of linear orders" (joint with 5 other authors), submitted.[3]

[AGK+19] "On cototality and the skip operator in the enumeration degrees" (joint with 6 other authors), Trans. Amer. Math. Soc., 2019.

Unfortunately, the information given about Prof. Soskova's contribution to the joint works is laconic and non-specifying – I did not find in the materials provided to me any information on the issue other than the statement made in the author's summary that in all joint papers the authors have equal participation.[4] The lack of more information is particularly embarrassing me given that half of the ten articles mentioned above have six or more authors each, and, strictly speaking, the results of the only non-joint one of the other five are not included in the dissertation (the spectra considered in the article in question consist of enumeration degrees, whereas the results for spectra indicated as contributions to the dissertation are for spectra of Turing degrees).

Regarding the way of inclusion in the dissertation of works from the above, I note that it is largely done by essentially direct copying of text. For example, from the very beginning (p. 50) of section 3.1 of the dissertation a copy of the text of the article [SS09a] begins – with some changes in fonts and, of course, with the necessary changes in the numbering of definitions, lemmas, etc., and in the conventional signs for the cited sources (the beginning of the original text is on the second page of the article). Definition 3.1.6 on page 54 of the dissertation deviates slightly from the corresponding settlement in the article, but then the similarity continues until the end of section 3.2 of the dissertation (page 58). Shortly afterwards, on page 59, a copy of the "Marker's extensions" section of the article begins and the repetition essentially continues until the beginning of page 68. Then on pages 71-73 we find text from pages 159-161 in the article [SS09b]. All things considered, out of the 25 pages of Chapter 3, about 20 turn out to be copied from [SS09a] and [SS09b] (even some typos from [SS09a] turn out to be repeated – I noticed such copies of typos on page 50, line 2 from below, line 65, line 5 from below, and page 67, line 7 from above). Extensive repetitions of text from articles are present also in the next chapters – texts from [CFH+18, KAV19, ACG+20, DHM+20, AGK+19] are repeated. In particular, I failed to notice any essential difference between the text of sections 4.2 and 4.3 in the dissertation (from page 79 to page 107) and the text of sections 2 and 3 in [CFH+18]. At that, in some places in the dissertation a second or fifth author is mentioned (see pages 84, 88, 145) without the reader being informed about the literal repetition of a text from a joint article (and the expressions "this work", "this paper" and "this article" from another of the articles are repeated without explanation in Chapter 6 on pages 155, 156, 206, 215 and 253).

I go now to a review of the scientific content and the most important in

[3] Not indicated where.

[4] It struck me, however, that in the thanks expressed on pages 20-21 of the dissertation several co-authors of papers out of the ten mentioned were omitted, namely R. Alvir, R. Miller, R. Kuyper, R. Weisshaar (despite the words "all my coauthors" available there).

my opinion scientific contributions in the dissertation. Its first two chapters are of a preparatory nature, and the contributions are in the next five. As these contributions are in fact from joint publications and I have no information about the specific contribution of Prof. Alexandra Soskova, I could not point it out and separate it from the total contribution of the authors.

The author's summary states that the original contributions of the dissertation are the answers given in it (respectively in Chapters 3, 4, 5, 6 and 7) to the following questions (except for the last one, they refer to structures with countable domains):

(1) How to define the jump of a structure as an analogue of the Turing jump in the degree structure $\mathcal{D}_T$ of Turing degrees? Are there any typical structural properties such as jump inversion theorems? Is the set of all jumps of the elements of the degree spectrum of a structure also a spectrum of another structure?

(2) Are there any model theoretical conditions that are sufficient for a structure to admit strong jump inversion?

(3) For the known effective codings of one class of structures into another class, is there an effective or more difficult decoding for some special classes as linear orderings and 2-step nilpotent groups, which are on the top of Turing computable embeddings?

(4) For any two copies of a computable order type, do their cohesive powers has the same order type?

(5) Are there any substructures with interesting properties in the degree structure $\mathcal{D}_e$ of the enumeration degrees, other than the total and the continuous degrees?

I agree with this statement, but with the natural stipulation that Prof. Soskova's share in each of the contributions in question depends on the number of relevant authors listed further in the author's summary.

Some of the significant contributions are the following:

Ch. 3 (the contributions are joint with Ivan Soskov). A jump of a structure is defined by considering its Moschovakis extension together with a new predicate, analogous to the Kleene set. Let $DS(\mathcal{A})$ (the spectrum of degrees of the structure $\mathcal{A}$) be the set of all Turing degrees of $\mathcal{A}$, and $DS_1(\mathcal{A})$ – the set of their Turing jumps. A jump inversion theorem is proved which is analogous to the classical Friedberg's jump inversion theorem and states the following (see p. 64 of the dissertation): if $DS\mathcal{A}) \subseteq DS_1(\mathcal{B})$ then there is such a structure $\mathcal{C}$ that $DS_1(\mathcal{C}) = DS(\mathcal{A})$ and $DS(\mathcal{C})) \subseteq DS(\mathcal{B})$. A generalization of the theorem with DS_n instead of DS_1 is also proved, where $DS_n(\mathcal{A})$ for $n \in \mathbb{N}$ is the set of the nth Turing jumps of the Turing degrees on $\mathcal{A}$. Some results of Slaman and Wehner are also generalized.

Ch. 4 (the contributions are joint with Calvert, Frolov, Harizanov, Knight, McCoy and Vatev). A structure $\mathcal{A}$ is said to admit strong jump inversion if it has the following property: a subset X of $\mathbb{N}$ computes the atomic diagram of a copy of $\mathcal{A}$, whenever the jump of X computes the atomic diagram of a copy of the jump of $\mathcal{A}$ (not all structures admit strong jump inversion). This chapter provides general model-theoretical conditions, which guarantee strong jump inversion of a structure (see page 80 of the dissertation). The obtained general result is skillfully applied to structures of some familiar classes, for example for some classes of linear orders and trees, as well as for Boolean algebras without 1-atoms (it is shown that an estimate of the complexity of an isomorphism is better for them than in the general case of low Boolean algebras). The general result also includes a result by Marker and Russell Miller. As a consequence, it turns out that the saturated model of the theory of differentially closed fields with characteristic 0 has a computable copy.

Ch. 5 (the contributions are joint with Knight, Vatev, Alvir, Calvert, Goodman, Harizanov, Morozov, Russell Miller and Weishaar). With Knight and Vatev, examples of Medvedev's irreducibility of graphs to linear orderingss and their jumps are given, and the reducibility of each graph to the second jump of a linear ordering is shown. Together with Alvir, Calvert, Goodman, Harizanov, Knight, Morozov, Miller and Weisshaar, Maltsev's result on interpreting a field in the Heisenberg group over it is significantly improved in two different ways. With Alvir, Knight and Miller, a result of Poizat about the interpretation of an algebraically closed field with characteristic 0 in the special linear group of matrices 2×2 with determinant 1 over it is improved.

Ch. 6 (the contributions are joint with Dimitrov, Harizanov, Morozov, Shafer and Vatev). The cohesive powers introduced by Dimitrov are an effective variant of the ultra-power model-theoretical construction, the role of ultrafilters being played in this variant by the cohesive sets of natural numbers (a set C of natural numbers is called cohesive if C is infinite, but some of the sets $C \cap W$ and $C \setminus W$ is finite, whenever W is a recursively enumerable set of natural numbers). The main results are as follows, where ω, ζ and η denote the order type of the natural numbers, of the integers and of the rational numbers, respectively, C is a cohesive set of natural numbers and $\Pi_C \mathcal{L}$ denotes the cohesive power of $\mathcal{L}$ over C for every computable copy $\mathcal{L}$ of ω:

(A) If $\mathcal{L}$ is a computable copy of ω that is computably isomorphic to the standard representation of ω then $\Pi_C \mathcal{L}$ has order type $\omega + \zeta\eta$.

(B) If $\mathbb{N} \setminus C$ is recursively enumerable and $\mathcal{L}$ is a computable copy of ω then the finite condensation of $\Pi_C \mathcal{L}$ has order type $1 + \eta$ (the finite condensation of a linearly ordered set is obtained, roughly speaking, by identifying those of its elements, between which there is at most a finite number of others).

(C) If $\mathbb{N} \setminus C$ is recursively enumerable then there is such a computable copy of $\mathcal{L}$ of ω that $\Pi_C \mathcal{L}$ has order type $\omega + \eta$.

(D) A more general result with a more complex formulation.

Ch. 7 (the contributions are joint with Andrews, Ganchev, Kuiper, Lamp, Joseph Miller and Maria Soskova). The results in this chapter refer to a substructure of the numbering degrees, namely the cototal degrees. These are the degrees of the cototal sets, i.e. of those that are enumeration reducible to their complements. As a tool in the research, the operator skip is used, where the skip of a set A is an uniform upper bound of the complements of all sets enumeration reducible to A. Many examples are indicated of enumeration degree classes that either guarantee or prohibit cototality. The complement of the graph of a total function is a cototal set, and the degrees that contain such a set are called graph-cototal. An enumeration degree is called weakly cototal if it contains a set that has a total enumeration degree. Graph-cototality implies cototality, and cototality implies weak cototality. The main result in this chapter is that the three properties are different. The construction of a cototal degree which is not a graph-cototal turns out to be especially difficult. A priority method with an infinite-injury argument is skillfully used for this purpose. A number of other interesting results have been obtained.

The application of Prof. Alexandra Soskova for the scientific degree "Doctor of Mathematical Sciences" is supported by an extensive positive reference letter from Academician Sergei Goncharov, Director of the Mathematical Institute "S. L. Sobolev" of the Russian Academy of Sciences.

Some remarks

The work [SS09a] is one of those used by Mrs. Alexandra Soskova in her habilitation as a professor, and is one of those mentioned in the current report on satisfying the minimum national requirements. This is not in accordance with a statement that logically follows from the wording of item 4 in the declaration submitted by Prof. Soskova, namely that she did not use her scientific papers indicated in the report in question to get the academic position of professor. However, a member of the governing body of FMI assured me that the text of the declaration is standard and the wording of the mentioned item 4 must be understood in a different way than the literal one and not leading to the mentioned contradiction.

On page 31, where enumeration operators are mentioned, it would be appropriate to cite a work by Uspensky, where essentially the same notion was introduced, namely the article "On computable operations" (Dokl. Akad. Nauk SSSR, **103** (1955), 773–776).

The name Łoś on page 159 of the dissertation and the name Fraïssé on pages 168 and 169 are not correctly presented (the first of these three inaccuracies occurs also in the manuscript [DHM$^+$20]).

Among the axioms at the beginning of section 6.3 of the dissertation, one of the first two is redundant (the same applies to the axioms at the beginning of Section 3 of [DHM$^+$20]).

The sentence after statement 7.4.12 has lost its meaning, because the sub-items in the mentioned statement are marked with 1., 2., 3. etc. instead of (a), (b), (c) etc., unlike the sub-items of the original (statement 3.15 in [DHM$^+$20]) from which it was copied.

It should be “Heisenberg” instead of “heisenberg” in the bibliographic data of the article [ACG+20].

It is not clear what 759, 783-784 and 376 mean in the data of articles [CCKM04], [DH15] and [Fro06]. In addition, the information “(Russian)” should be added to these data, as well as to the data of other articles in Russian journals, if not provided.

I do not see why an inversion in the name “A. Soskova” was needed in the data of the article [KAV19].

In the data of the article [Lav63], “I. S. Lavrov” should be replaced by “I. A. Lavrov”, and “2:5” - with “2:1, 5”.

Judging by the number of the last page, “I, II” should be replaced with “I” in the data of the article [Mos69].

Instead of “nicht-charakterisierbarkeit” and “aussagen” it should be “Nicht-charakterisierbarkeit” and “Aussagen”, respectively, in the data of the article [Sko34] (the error is also present in the bibliography of [DHM+20]).

In the data of the article [Stu10], it would be better to read “[Stu09]” instead of “mr2586684”.

The title of the dissertation [Vau55] is formatted not in the same style as the other titles of dissertations included in the bibliography.

In the data of the article [Mat70] given in the abstract, instead of “Ju.” and “Matijasevič” it should be, respectively, “Yu.” (or “Yuri” as in data of the next source) and “Matiyasevich”.

Conclusion

The presented dissertation is a meaningful and detailed exposition of numerous results in an important modern field of the theory of computability, the production of which required perfect knowledge of the subject and remarkable technical skill. Most of these results have been published in scientific editions with great international prestige and have received a serious response in the international scientific community. However, it is not a trivial question whether on the basis of this work Prof. Alexandra Soskova can be awarded the degree for which she claims. The main problem is that, as the information given by Prof. Soskova about the authorship of the results shows, it is largely shared between her and many other researchers, and the regulations on the basis of which we proceed do not give clear enough instructions on how to proceed in such cases. However, given the abundance and quality of these results, their difficulty, the prerstige of the editions where they are published, and the response the results have received in the scientific community, I think that despite the problem in question and some minor ones seen in the review, there are sufficient grounds to award Prof. Alexandra Soskova the scientific degree ”Doctor of Mathematical Sciences”. I propose that the esteemed scientific jury adopt awarding it.

Sofia, March 16, 2021 Reviewer’s signature: