REVIEW REPORT

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Related to the procedure of a competition announced in Bulgaria's State Newspaper \ДВ No 21 from 13. 03. 2020\ for an academic position of Professor in
The Faculty of Mathematics and Informatics at
Sofia University "St. Kliment Ohridski"
in scientific area 4.5. Mathematics (Operations Research).

In the procedure of this competition one candidate applied: Associate Professor DSc Nadia Peycheva Zlateva. The submitted documents are complied with respect to the requirements of the Law for Academic Staff Development in the Republic of Bulgaria, the Regulations to the Law, and the Regulations on Terms and Conditions for Acquiring Scientific Degrees and Holding Academic Positions at Sofia University "St. Kliment Ohridski"

Nadia Zlateva graduates in 1996 at the Faculty of Mathematics and Informatics at Sofia University "St. Kl. Ohridski". In 1999 she achieves PhD in specialty of Operations Research. In 2018 Nadia Zlateva defended DSc dissertation. She has been an associate professor at the Faculty of Mathematics and Informatics since 2004.

The total number of scientific publications of Nadia Zlateva is 29. There are 4 more general publications and two dissertations as well. From this list of publications 8 are presented at this competition and none of them has been presented at a previous competition. These 8 are joint publications with one coauthor. All of them are published in prestigious journals with impact factor. To mention the journals: Proceed. Amer. Math. Soc., J. Convex Anal., JOTA, and Compt. Rend. l'Ácad. Bulg. Sci. (Доклади БАН).

The number of citations is 290 with 16 of them in dissertations and habilitation monographs. The papers presented at this competition have 5 citations. It is to be mentioned that 5 of the 8 papers are published recently, i.e. in the period 2017-2020.

The scientific activity of Ass. Prof. Nadia Zlateva includes participation in 16 scientific and education-scientific projects, 3 long-term specializations in France, and more than 15 presentations at international conferences.

Her lecturer activity is very rich as well as her scientific administrative activities. Nadia Zlateva has been a Head of the Cathedra of Probability, Operations Research, and Statistics in the Faculty of Mathematics and Informatics during the period 2008-2012

and Vice Dean in 2011-2017. Her expert activities include referring scientific papers and projects for three consecutive sessions of The National Fund for Scientific and Technological Development of the Chilean National Commission for Scientific and Technological Development. She is a member of an expert group for program accreditation at the Technical University of Sofia since 2008.

The results presented by Nadia Zlateva at the completion are in the area of Operations Research and can be split in three thematic areas:

- Perturbation spaces for minimization of integral functionals,
- Surjectivity of mappings in Frechet spaces,
- New proofs of theorems in the area of variational analysis.

The theme of the first field of research is covered in the papers [2б] and [3б]. In both works an infinite-dimensional integral functional is subject to minimization, whereas in [2б] the sub-integral function is convex and in [3б] it is given in a more general form. A perturbation space is to be constructed such that when the sub-integral function is perturbed by a suitable function from this space the minimization problem has to possess a solution.

In [2б] the convex case is treated and the convex sub-integral function is defined in a Banach space which is not necessarily reflexive. A general perturbation method is presented and the perturbed problem appears to be in the same form as the original one while the perturbed function can be chosen to be arbitrary close to the original. The main result gives a new variational principle.

Further line of research includes the work [3б], where the sub-integral function is not convex. In this setting an idea initially proposed by Deville, Godfroy and Zizler is developed in order to obtain a new variational principle. The axioms on the perturbation space are more specific in comparison to [2б] but according to the authors these axioms are easier to verify and are also sufficient for many applications.

The papers [5б] and [6б] are devoted to the problem of surjectivity of mappings in Frechet spaces. This line of research originates from a theorem of Nash-Mozer which generalizes the inverse mapping theorem in Banach spaces. The Nash-Mozer theorem arouses enormous interest mainly due to its applications, for instance in the analysis of local solutions of partial differential equations. The Frechet spaces are convenient for the study of nonlinear systems with infinitely smooth set of data. They are linear topological with translation invariant metrics which is usually defined by a sequence of semi-norms and thus they are close to Banach spaces in a certain sense. Various families of infinitely smooth functions can be thought of as Frechet spaces. The main difficulty in proving theorems of Nash-Mozer type is the mapping surjectivity question. In a paper of Ekeland, this obstacle is overcome by requiring Gateaux differentiability and after applying Ekeland’s variational principle.

The paper [5б] gives a result about surjectivity of set-valued mappings. This result is obtained by semi-norm estimations. Instead of Gateaux differentiability, a modification of the notion of contingent derivative of a set-valued mapping is used. The connection between Nash-Mozer theorem and the metric regularity is underlined. It has been proved also that the original assumptions in Nash-Mozer theorem imply metric regularity in a weak form.

In [6б] a simple proof of a partial case of Nash-Mozer theorem is presented. Under some mild restrictions but still general enough for many applications, the semi-norms estimates from an Ekeland's paper are improved.

The papers [1б], [4б], [7б], and [8б] contain new proofs of theorems in the area of the variational analysis. Research like this is interesting not only for educational purposes but also for the possibility to bring new points of view to known facts which sometimes lead to the development of new methods and techniques.

The work [1б] brings a new proof of Moreau-Rockafellar theorem according to which a proper convex semi-continuous from below function defined in Banach space is characterized uniquely up to a constant by its subdifferential. The proof in [1б] follows a rather classical idea for Riemann integrability of a monotone function. The first proof by Moreau is in Hilbert space and makes use of Moreau-Yosida regularization, and the Rockafellar's proof involves duality arguments. There are many published proofs of this theorem and in a lot of them the directional derivative is approximated which reduces the problem to the one-dimensional case. An alternative approach involves the mean-value theorem of Zagrodny.

In [4б] a new proof of the classical fact that the subdifferntial of a convex function is monotone has been proposed. The theorem of Rockafellar has been proven first by Minty in case of a Hilbert space with the help of Moreau-Yosida regularization technique. The complete proof by Rockafellar in arbitrary Banach space demonstrates that Fenchel conjugacy arguments can be used in non-reflexive spaces. The proof given in [4б] makes use of Minty type minimality condition.

The work [7б] brings a new proof of a result of Frankowska according to which the metric regularity of a set-valued mapping is characterized by its contingent variation. The contingent variation is a concept generalizing the notion of contingent derivative. The proof in [7б] gives global characterization while Frankowska's result describes the local modulus of regularity.

New method of proof of a theorem of Correa-Jofre-Thibault that the monotonicity of the subdifferential of a semi-continuous from below function implies its convexity is given in [8б]. This theorem generalizes a known fact from the classical analysis in the case of differentiable functions. Main tool of the various proofs in the general case is Zagrodny's mean-value theorem. In [8б], the proof employs barrier functions and the

concerned subdifferential is axiomatic which gives a slightly more general result than the one by Correa-Jofre-Thibault.

In conclusion, I have no doubts in the personal contribution of Ass. Prof. Dsc Nadia Zlateva to the scientific results presented at this competition and confirm my **positive opinion** with respect to her candidacy for **the academic position Professor at the Faculty of Mathematics and Informatics at Sofia Univesity "St. Kliment Ohridski"**. From the submitted documents which comply with the Law for Academic Staff Development in the Republic of Bulgaria, the Regulations to the Law, and the Regulations on Terms and Conditions for Acquiring Scientific Degrees and Holding Academic Positions at Sofia University "St. Kliment Ohridski", as well as from my personal observations on the entire scientific, educational, expert, and scientific-administrative career of **Ass. Prof. Nadia Zlateva**, I am convinced that **she fully deserves the position of Professor at the Faculty of Mathematics at Sofia University "St. Kliment Ohridski" and therefore give recommendation to the Scientific Jury and the competent organ of the Faculty of Mathematics and Informatics at Sofia University "St. Kliment Ohridski" on the choice of Ass. Prof. Nadia Peycheva Zlateva for the academic position Professor in the area of Mathematics (Operations Research)**.

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/Prof. Dr. Nikolay Zhivkov/