

REPORT

**on the competition for academic post of “Associate Professor”
in professional field 4.5. Mathematics, scientific speciality “Differential Equations”,
for the needs of Sofia University “St. Kliment Ohridski” (SU),
Faculty of Mathematics and Informatics (FMI),
announced in SG, issue 65/ 16.08.2019 and on the Website of FMI and SU**

This report is prepared by **Prof. D.Sc. Virginia Kiryakova**, Institute of Mathematics and Informatics (IMI) – Bulgarian Academy of Sciences (BAS), a member of the Academic Board on this procedure according to Order № ПД 38 - 593 / 11.10. 2019 г. of the на Rector of Sofia University, appointed as assessor at the first meeting of the Board.

For participation in this competition there applied **only one candidate: Senior Assistant PhD Tsvetan Dimitrov Hristov** (Fac. of Mathematics and Informatics – Sofia University “St. Kliment Ohridski”).

I. General description of the presented publications and documents:

1. Data for the procedure

The documents presented for the competition by the applicant satisfy the requirements of the Law Act for Development of Academic Staff in R. Bulgaria (abbrev. further as the “**Law**”, of 26.02.2019), the (National) Regulations for its applications, and the Regulations of SU “St. Kl. Ohridski” for the conditions and rules for obtaining scientific degrees and occupation of academic posts (abbrev. as “Regulations of SU”).

For participation in the competition, the applicant **Sen. Assist. PhD Tsvetan Hristov** has presented a list of 14 scientific works, 13 of which are publications in Bulgarian and foreign scientific editions and forums, and 1 is educational manual on the topic of the competition. He has presented also **all** required documents (official notices, certificates for working experience, references, recommendations, etc.), supporting the applicant’s achievements and previous positions and experience.

2. Data for Applicant

The applicant Tsvetan Hristov had his higher education on Mathematics with Master’s diploma, speciality “Differential equations” in 1998 from Sofia University, Fac. of Mathematics and Informatics (FMI), and since then he works there, for more than 19 years. In 2006 he defended a PhD thesis on the subject “Singularities of the solutions of hyperbolic equations in domains with characteristic boundary”. Dr. Ts. Hristov has occupied the academic posts of “Assistant” and afterwards “Senior Assistant” in FMI since 2005. In the period 2002-2010 he has scientific specializations in Germany, Italy and France.

3. General characteristic of the scientific works and achievements of the applicant

The author’s reference note (Annotation, Doc. # 13) reflects fairly the applicant’s scientific contributions in the presented works. The research interests of Dr. Ts. Hristov, yet from the time of his Master’s thesis, afterwards in the PhD thesis, and up to now as well as his contributions, are in the scientific speciality of the announced competition for the post of Asso. Prof., namely – “Differential equations”. In particular, he studies 3-dimensional and 4-dimensional boundary value problems for degenerate hyperbolic PDE of the type of Tricomi and Keldysh, in analogy with the mul-

ti-dimensional problems considered by Protter and describing about sound processes of the gas dynamics which plane analogues have been studied also by Morawetz, Lax and Phillips.

In my opinion, the applicant's contributions in the presented works can be characterized as enrichment of some existing knowledge combined with using and introducing new hypotheses and methods. These confirm the applicant's abilities to upgrade former theories and characterize him as already established in his field scholar.

- a) The presented scientific publications satisfy the national minimal requirements (Item 26, # 2 and 3 of the Law) as well as the additional requirements of Sofia University for occupation the academic post of Associate Professor in the speciality of the announced competition, namely:
 - The applicant holds the MSc degree and PhD degree (obtained in 2006, on same subject);
 - He has working experience in FMI – SU for more than 19 years, of which 15 years (since 2005) as lecturer at the posts Assistant and Senior Assistant.
 - All his publications, as well as the other related activities reported in the documents for the application are well reflected (provided reference) in the Sofia University electronic system "The authors". I personally do not have access to authors' citations there, but these and all other data are introduced in the system.
 - All minimal national requirements (science metric data and indicators) for the academic post of Associate Professor are fully satisfied, as seen in the TABLE provided further in this report.
- b) The presented publications of the applicant do not repeat and do not include any of the works used for previous procedures on obtaining science degree and academic posts.
- c) There is no plagiarism found in the works presented for the procedure.

4. Characteristic and evaluation of the teaching activities of the applicant

The candidate Sen. Assist. Tzvetan Hristov possesses a long-year and rich teaching experience in FMI – SU. It is evaluated as excellent in the provided reference letters by Prof. N. Popivanov and Asso. Prof. G. Dachev, as well as according to impressions of other known colleagues from his Faculty.

He has taught lecture courses on Calculus 1, Calculus 2, Differential Equations (DE), PDEs, DEs and applications, Selected chapters of Analysis, and also seminar exercises for different Master and B.Sc. programs in FMI, including also in English.

On the speciality of the competition, the applicant has one student with defended Diploma thesis, has a series of prepared curricula of courses, published guidance educational books and manuals, participated in many research projects oriented also to practical applications. He has been also a Leading researcher of a Faculty's Project on the European program for improving the quality and efficiency of the education process.

5. Analysis of the scientific achievements of the applicant in the works presented on the competition

In the research works, presented for the competition, there are studied multi-dimensional (3D and 4D) boundary value problems for degenerate hyperbolic differential equations. Similar problems have been formulated by M. Protter (Berkeley University) in the 50s of 20th century, as multi-dimensional analogues of the classical Darboux problems in the plane. The applicant provides a formulation also of a more general problem for elliptic-hyperbolic equations, which is a multi-dimensional analogue of the 2D Goursat-Guderley-Morawetz problem, arising from applications for about sound currents of fluids. It happens that the new problems are essentially more complicated

and rich of many new effects in comparison with the plane analogues. Aziz and Schneider proved a result for uniqueness for 3D equation of mixed type, but the question for existence has not been solved by today. In the case of hyperbolic-parabolic equations, the Protter problems are incorrect - the homogeneous adjoint problems have infinity-dimensional kernels of the classical solutions. For that reason, it is necessary to look for generalized solutions in special functional spaces with weights.

It happens that they exist solutions with strong singularities at one point, even for well smooth right-hand sides.

Specially, I like to emphasize on the important role of the systems of special functions used in all these author's studies. To this aim, it is enough to mention that after a suitable reductions of the Protter problem to a 2D problem (depending on the dimensionality of the space), the corresponding Riemann function for the wave equation, as well as the Riemann-Hadamard function for the degenerate ones, is written down by means of hypergeometric functions – the Gauss functions and the more general ones, by using their subtle properties. The derived results are based on integral representations of the solutions in terms of these special functions. My opinion is that this is important and useful because namely in the special functions involved in the representation's kernel are encoded the singularities of the solution.

1. In papers [3], [4] and [5] Protter's problems are studied for 3-dimensional degenerate hyperbolic equations of Tricomi type:

$$t^m(u_{x_1x_1} + u_{x_2x_2}) - u_{tt} + b_1u_{x_1} + b_2u_{x_2} + bu_t + cu = f, m > 0. \quad (1)$$

Suitable classes of quasi-regular solutions are defined and uniqueness results are obtained. There are found the coefficients such that the adjoint problems to have unique quasi-regular solution. Generalized solutions are introduced that can have a singularity at one point of the domain's boundary. Under suitable conditions for the coefficients, there are obtained results not only for uniqueness but also for existence of solution of 3rd boundary problem of Protter P_α . For each $n \in \mathbf{N}$ there are found smooth right-hand sides $f_n(x, t)$, and conditions on the coefficients under which the generalized solutions have at least a power singularity at the origin: $|u_n(x, t(|x|))| \geq |x|^{-n} |\cos(n \arctan(x_2 / x_1))|$.

2. In papers [4] and [6], a formulation of a new problem of Protter PK is proposed for 3D weakly-hyperbolic equations of Keldysh type:

$$u_{x_1x_1} + u_{x_2x_2} - (t^m u_t)_t + b_1u_{x_1} + b_2u_{x_2} + bu_t + cu = f, 0 < m < 2. \quad (2)$$

In contrary to the Tricomi case (1), in equation (2) the degeneration happens in front of the higher derivative on the time variable. Because of this reason, the formulation and the study of boundary value problems for equations of Keldysh type are essentially more complicated. The works and results on this difficult topic are comparatively less in quantity while the interesting and unsolved problems are much more. In the considered problem PK, similarly to the elliptic-parabolic case, no conditions are prescribed upon the parabolic part of the domain's boundary but only on a part of hyperbolic boundary. The applicant finds infinitely many classical solutions of the homogeneous adjoint problem, that is the problem PK appears incorrect. With the help of Hardy-Sobolev inequality, an elegant evaluation is made in order to prove uniqueness in a special class of quasi-regular solutions.

In the papers [7] - [9] and [12] generalized solutions of the problem PK are introduced. These solutions can have a singularity at the origin, and for the derivative u_t it is allowed to have singularity up to a given order on the whole parabolic part of the boundary. For equations without lower order derivatives, results for existence and uniqueness are obtained and it is established that when the right-hand side is a trigonometric polynomial of order l , the solution may have at most a power singularity $|x|^{-l}$. In the more complicated case of equation with lower order terms, results are obtained for

existence and uniqueness for order of $m \in (0,1)$. There are found smooth right-hand sides and conditions on the coefficients for which the generalized solutions have at least power singularities.

3. In the papers [1, 2] and [10, 11] a problem PK for 4D equations of Keldysh type is studied:

$$u_{x_1x_1} + u_{x_2x_2} + u_{x_3x_3} - (t^m u_t)_t = f.$$

It is proved that this problem is also incorrect, because it has infinitely-dimensional co-kernel of classical solutions. A generalized solution is defined in a suitable weighted space. With the help of special functions – namely, Appell’s hypergeometric series, a Riemann-Hadamard function is constructed. In this way an integral representation of the generalized solution is obtained which includes a sum of suitable hypergeometric functions in the kernel, and theorems are proved for existence and uniqueness in the class of the introduced generalized solutions admitting singularities. When the right-hand side $f(x,t)$ is a harmonic polynomial on the spherical harmonics, an a priori estimate is proved which exhibits the maximally possible singularity of the solution at origin. The asymptotic behaviour of the generalized solution is found at the singular point. Let us again emphasize on the role of the used special functions. By means of heavy tedious evaluations, conditions for orthogonality are written down. When a different number of these conditions are fulfilled, the solution possesses singularities of different orders.

4. It makes a good impression that the applicant has prepared 2 chapters of educational manual, and he has been leading coordinator and/ or participant in several projects aiming improving the efficiency of education on differential equations in FMI. There an educational software is developed for computer simulation and visualization of real processes modelled by means of differential equations. For example, the important practical problem for atmosphere pollution with a volcanic ash is simulated. The applicant has shown that along with deep knowledge on differential equations, he can explore also numerical methods and various computer algebra systems for scientific calculations and visualization. He has been also advisor for a successfully defended MSc thesis in this field.

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Citations/ Impact of candidate’s results: The presented publications have received about 60 citations in editions with impact factor (IF) or impact rang (IR), as follows: 10 citations in journals with IF, and 50 citations with IR (in *AIP CP*). The observed citations in papers with IF are, respectively, in: *J. Math. Anal. Appl.* Q1– 2 items, *Integr. Transf. Spec. Funct.* Q2 – 2 items, *Adv. In Math. Phys.* Q3 – 3 items, *Diff. Eqs.* Q4 – 1 item. Thus, the calculated points for Indicator $\Delta 11$ are $60 \times 8 = 480$ points, instead of the required 50 points.

The mentioned citations come from recognized foreign authors in the field (E. Moiseev, J. Mauersberger, I. Egorov, V. Fedorov, K. Zhang, M. Schneider, G. Dildabek, M. Saprygina, M. Sadybekov, A. Kholomeeva, J. Yakovleva), and others are from the group of Bulgarian colleagues as N. Popivanov, T. Popov, A. Nikolov, we have in mind items without self-citations.

Joint publications/ Personal contributions: Of the presented 13 works for the competition: 3 are single-authored by Ts. Hristov, 10 are jointly authored (with Bulgarian colleagues of same group – N. Popivanov, A. Nikolov, and also with M. Schneider). I assume an equal participation of Dr. Ts. Hristov in the joint publications and the contributions there.

Approbation of the results: Dr. Tsvetan Hristov has presented his results at a series of national seminars and forums (18) and in 36 talks at international conferences – in Bulgaria, and abroad - in Portugal, Russia, Czech R., Norway, Greece, Slovakia, Germany, N. Macedonia. His paper at the School for Young Scientists in Nalchik (Russia) in 2010 has been awarded with a prize for scientific novelty and originality of results.

Numerical indicators:

According to the Sofia University regulations and the regulations for the Law application (*revised and upgraded, SG issue 15 / 19.02.2019*), on the minimal national criteria for occupation of the academic post “Associate Professor”, **the required indicators and the indicators for the applicant Tsvetan Hristov are provided in the TABLE below. It is evident that these criteria are satisfied, and for Γ and Δ are even essentially over-satisfied:**

Groups of indicators	Content	Minimal requirements for Asso. Prof.	Indicators of Sen. Assist. Dr. Ts. Hristov
A	Indicator 1 (PhD thesis)	50	50 (defended 2006)
B	Indicator 3 (Habil. thesis / equiv. publs.)	100	111 (1 - in Q1, 1 - in Q4)
Γ	Indicator 7 (Sci. publ. indexed in WoS or Scopus)	200	312 (10 - in Q4, 8 - with SJR; 1 – not indexed)
Δ	Indicator 11 (citations, ...)	50	480 (60 cits.: 10 – with IF, 50 – with SJR)
E	Indicators 12 - 20	- (Not required)	Research Projects – total 23 (with Bulg. Nat. Fund, SU, bilateral with Russia, etc)

6. Critical Notes and recommendations

The documents, necessary references and the publications presented by Dr. Ts. Hristov for participation in this competition are carefully prepared. There are just few typos observed (mainly for the name of Tricomi) which are unavoidable in such a big volume of stuff.

My recommendation to the applicant is after the habilitation to direct his efforts toward more self-authored publications and mainly in specialized journals with impact factor (as such publications the applicant had more after PhD defense).

7. Personal impressions

I have known the candidate Ts. Hristov from discussions we had on his research topics and from his talks presented at the international conferences „Applications of Mathematics in Engineering and Economics (AMEE)” and other forums in Bulgaria. His presentations on the results for this competition have met approvals by the participating foreign experts. As a lecturer, he showed good knowledge on the subject of the works, ability to communicate efficiently and to present convincingly his results.

8. Concluding notes:

After I checked the stuff and scientific works presented by the applicant for the competition, and after analyzing the importance and correctness of the results as contributions, **I can confirm that his scientific achievements and science metric data satisfy the requirements** of the Law, Regulations for its application and Regulations of Sofia University **for occupation of the academic post “Associate Professor”** in the professional field (Mathematics) and scientific speciality (Differential equations) of the competition announced.

In particular, the applicant satisfies all minimal national criteria in the professional field (see TABLE) and no traces of plagiarism are found in the presented works.

Hereby, I give my **POSITIVE EVALUATION** for the candidate on this procedure.

II. GENERAL CONCLUSION:

According to all above said, I am confident to **recommend to the Academic Board** to propose to the body of Faculty of Mathematics and Informatics – Sofia University, competent on the election on this procedure, **to elect Senior Assistant PhD Tsvetan Dimitrov Hristov to occupy the academic post “Associate Professor”** in the professional field 4.5 Mathematics (Speciality “Differential equations”).

December 8, 2019

Author of the Report and Member of the Academic Board:

(Prof. D.Sc. V. Kiryakova)