REVIEW

By Assoc. Prof. Boyan Georgiev Zlatanov, PhD
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In reference with a dissertation thesis

for awarding the Educational and Scientific Degree "**Doctor**" in
Area of higher education: 4. Natural Sciences, Mathematics and Informatics;
Professional field: 4.5. Mathematics;
Doctoral program: Mathematical analysis.

Dissertation title: On Some Diophantine Equations and Inequalities
Author of dissertation: Zhivko Hristov Petrov

Supervisor: Prof. Doychin Ivanov Tolev, Doctor of Sciences

GENERAL DESCRIPTION OF THE PRESENTED MATERIALS

The presented publications satisfy both the national minimum requirements and the specific requirements of the Faculty of Mathematics and Informatics of Sofia University "St. Kliment Ohridski" for a PhD degree.

GENERAL CHARACTERISTIC OF DISSERATION

The dissertation consists of 93 pages divided into 5 chapters (Introduction, Auxiliary Statements and the last 3 chapters are new results obtained by the author), and a bibliography of 85 titles. Three publications are included in the dissertation. Two of these articles are printed in journals with an impact factor (Proceedings of the Steklov Institute of Mathematics IF = 0.623, Q3; Monatshefte fur Mathematik, IF = 0.735, Q2). The third one is single authored and it is published in Annual of Sofia University, (indexed in MathReview). The first publication is cited by W. Zhu in Representation of integers as sum of fractional powers of primes and powers of 2, Acta Arithmetica, 181, 2, (2017), 185-196, IF = 0.476, Q4.

Theorems and hypotheses that are related to the research of the dissertation are formulated in Chapter 1.

The results, which are used in the proofs of the new results in Chapters 3, 4 and 5, are presented in Chapter 2.

Chapters 1 and 2 provide a comprehensive overview of the results obtained so far.

Chapter 3 deals with a classical task from the analytical number theory, namely, to prove that for sufficiently large natural numbers N the equation $[x_1^c] + [x_2^c] + \cdots + [x_n^c] = N$ has a solution, where $c > 1$ is not a natural number, N is a sufficiently large natural number and $[t]$ denotes the integer part of t. The author has investigated the above mentioned task in a particular case: the existence of a sufficiently large natural number N, so that the equation

$$[p^c] + [m^c] = N$$

has a solution for any $1 < c < \frac{29}{28}$, where p is a prime number and m is almost prime with at most $\left[\frac{52}{29-28c}\right] + 1$ prime factors.

The existence of a solution of an equation with prime numbers of a special kind is investigated in Chapter 4. It is proven that for sufficiently large natural numbers N, the equation

$$[p_1^c] + [p_2^c] + [p_3^c] = N$$

has a solution in prime numbers p_1, p_2, p_3, such that any of the numbers $p_1 + 2, p_2 + 2, p_3 + 2$ has at most $\left[\frac{95}{17-16c}\right]$ prime factors, counted with multiplicity, provided that $1 < c < \frac{17}{16}$.

The author has proven several inequalities connected to the Piatetski-Shapiro inequality. Namely, he has obtained sufficient conditions for the existence of a solution in prime numbers of the inequalities

$$\left|\sum_{k=1}^{s} p_k^c - N\right| < (\log N)^{-1}$$

when $s = 2, 3, 4$ or $s > 4c \log c + \frac{4c}{3} + 10$.

The proofs of the mentioned six new results use classic techniques of the analytical number theory. Of course, every classic technique needs to be applied in a particular way, providing different challenges to be overcome, which is done by the author. Better estimates of known constants are found. The proofs are non-trivial, difficult, long and many precise estimates are needed. New auxiliary results are obtained, that are interesting in themselves. In my opinion, in the proof of Theorem 3.1.1, a new technique is applied in estimating the sum $W(v)$ by presenting the sum as a linear combination of similar sums and an appropriate definition of the function $g_z(t)$

I accept that the authors in the publications are listed in alphabetical order and have an equal contribution.

CONCLUSION

My evaluation of Zhivko Hristov Petrov's PhD thesis, publications and contributions is positive.

The presented PhD thesis corresponds completely to all the requirements, conditions and criteria of the Law of Development of the Academic Staff in the Republic of Bulgaria, the Rules for the implementation of the above mentioned law, the Rules for the conditions and order for acquiring academic degrees and academic positions at Sofia University "St. Kliment Ohridski", as well as the specific requirements of the Faculty of Mathematics and Informatics at Sofia University.

On the basis of all written above, I am convinced in my **positive evaluation** and I would like to make a recommendation to the members of the honorable scientific jury to award the Educational and Scientific Degree "Doctor" to Zhivko Hristov Petrov in the area of higher education: 4. Natural Sciences, Mathematics and Informatics, Professional field: 4.5. Mathematics; Doctoral program: Mathematical analysis.

19.06.2019   Signature:
Plovdiv   /Assoc. Prof. Boyan Zlatanov, PhD /