Referee report for the PHD thesis of Mira Izak Bivas

"Differential Inslusions with Nonconvex Right-hand Side"

The differential inclusions in which the set-valued mapping involved is not convex-valued are notoriously difficult to tackle. To illustrate that statement, I would use the following example. Imagine a flow of a river which has a number of big rocks in the middle and also with little and big islands scattered along together with some peninsulas that encircle little bays. Also, imagine the flow without rocks, islands and peninsulas. It is clear that it is much more difficult to describe the flow in the former case than in the latter.

In the thesis, Chapter 1 is the introduction, and Chapter 2 gives the necessary background. In Chapters 3 and 4 contain the main results of the thesis. in Sections 3.1 and 3.2 the so-called viability problem is studied in increasing generality. The analysis is based in the introduction of special invariant ε-approximations of the original right-hand side, with upper semi-continuous convex-valued mapping satisfying the Carathéodory condition. The trick is, very roughly, to prove local existence for the approximation and then "glue" the pieces to get a solution of the original inclusion. The more general case when the pocal partition is not finite is also covered. While Section 3.3 contains known results, the main thrust in Section 4 is to show the existence of a solution of a differential inclusion the values of which belongs to a given set (viable solutions) and the right-hand side satisfies a Carathéodory condition of mixed type; this covers a number of known results.

In Chapter 4 is devoted to studying "sweeping" processes (according to J. J. Moreau) in which the right-hand side of the inclusion is described by a (limiting, hence nonconvex) normal cone mapping to a nonconvex moving set which is not regular (the proximal normal cone is different from the Clarke normal cone) but admits a so-called regular Whitney stratification (a large class of constructively definable sets possesses this property). The associated (nonautonomous) projection process is also considered and existence results for both problems are obtained. An application of the obtained results is presented to crowd motion modeled by a finite number of movable objects that are definable in some o-minimal structure, as well as immovable obstacles. Besides an existence result, the authors give a counterexample showing that for a highly nonregular (Cantor-like) set (even independent of time) the projection process may not admit a solution, while the respective sweeping process has one.

According to my knowledge, the results are new and mathematically correct.

According to my experience in supervising and evaluating a number of theses and applications for promotions, the thesis of M. Bivas fully satisfies the requirement for obtaining a PhD degree, e.g., in a first tier university in the US as well as universities in Bulgaria and other European countries.

There are some statements I would consider questionable, which, however, are not serious enough to devaluate this thesis. Still, I would like to mention them.

For example, I do not fully agree with the statement that the differential inclusions "were born and developed as a mathematical curiosity...". The issue is a bit different: the set-valued mappings were not considered worth studying (and some people still think like that). In his review to a book the late Jon Borwein noted that multivalued mappings have been avoided as much possible until the 1970s. He cites Bonsall's review published in Proc. Edinburgh Math. Society, vol, 13, 1963, of the book of C. Berge, Espaces topologiques: Fonctions multivoques, Dunod, Paris 1959, now classics; here is a part of it:

"It is difficult to detect a consistent purpose behind the writing of this book, or a substantial class of readers to whom it is intended. The first half of the book is in some respect an excellent introduction to general topology, and I particularly like its thoroughness over elementary matters and its unusually explicit use of quantifies. On the other hand, its utility for the beginner is surely greatly reduced by the author's insistence on allowing functions to be many-valued."

Also, on p. 2 it is claimed that "convexity is required for the multi-valued fixed point theorem". This of course is not true, there are fixed point theorems for set-valued mappings acting from a complete metric spaces to itself.

There are two figures, Figure 2.2 and 2.3, that are copied from the book "Variational analysis" by Rockafellar and Wets; this should have been mentioned. The name of the author of reference [27] is misspelled: it should be St. Lojasiewicz, Jr., the son of the famous Stanislaw Lojasiewicz whose name is connected with a basic inequality in algebraic geometry. And finally, the bibliography should have been written in an unified way.

Mira Bivas is currently a PhD student at the Department of Mathematical Analysis, School of Mathematics and Informatics, the Sofia University "Kl. Ohridski". She graduated from Sofia mathematical high school, probably the most prestigious school for mathematically gifted children in Sofia. Then she obtained Baccalaureate and Master degrees at the Sofia University and immediately after that was accepted to the doctoral program at the same university.

In her short but impressive career, Mira has published four papers, one of which at SIAM Journal on Control and Optimization, the highest ranked journal in the field of her studies. Her research has been funded by the research foundation of the Sofia university where she has participated in several projects.

Mira has reported her mathematical results at several conferences and scientific seminars. I would like to focus on one such event where I was also a participant. I am involved in a research project with a group of people at the Aerospace Engineering Department of the University of Michigan. In November 2016 I invited Mira, together with her supervisor, Prof. Nadezhda Ribarska, to visit me in Ann Arbor. Before their arrival, I mentioned to my colleagues that two Bulgarian mathematicians would visit me and, as a result, Mira was invited to give a talk at the student seminar of the Aerospace department. The name "student seminar" is a bit misleading; it is actually a seminar where PhD students, postdocs and professors of all ranks actively participate to vigorously discuss current advances in the broad field of control. I cannot miss to mention that this department is the highest ranked among aerospace departments in the US.

Mira gave a brilliant talk at the seminar, which impressed the audience and provoked many questions. At that seminar Mira acted as a mature scientist who has a clear perspective to her research area, can convincingly present her results, and is able to answer difficult questions.

I strongly recommend to the PhD committee to grant Mira Bivas the title "Doctor".

Dr. Asen L. Dontchev

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