# The Lucas Paradox, Technology Waves and Structural Characteristics: An Open Economy Growth Model

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#### Abstract

How does a foreign direct investments affect economic growth? The direction and size of the gains from FDI shape our understanding of the benefits of globalization. In this paper we analyze a model of exogenous growth which is based on an open economy, cyclical technological behavior and FDI. The unique property of the model is that it assumes that FDI are connected with a leading technological sector, structural characteristics of the economy and long-run growth patterns. Those factors then jointly determine the production level of the economic system. For simplicity only one sector exists and only one good is produced at any given time. Further we look at a certain variations of the proposed model, which includes imperfect technology absorption.

Keywords: FDI; growth; innovation; technology waves

# 1 Introduction

The growth rate of an open economy roughly depends on four distinct forces: the growth rate of technologies, the international capital flows, the international labor movements and structural characteristics. Over the last three decades, international capital flows, which we will call foreign direct investments (FDI), have become increasingly important for developing countries. According to the neoclassical growth models, the effect of FDI on growth is identical to that of domestic investments, and they should flow from rich to poor countries. In reality, that is not always the case - this observation is known as "The Lucas Paradox" (Lucas, 1990) - while there are indeed some rich countries that invest in poor ones, why investors are restraining themselves, from investing in countries like Burundi, Niger or Malawi? A number of solutions to the Lucas paradox have been proposed in the literature (Jiandong and Shang-Jin, 2006) : first - thinking of a worker in a rich country as effectively equivalent to multiple workers in a poor country, second - adding human capital as a new factor of production, third allowing for sovereign risk, and fourth adding trade costs. In the following model we will propose an additional solution the connection between FDI, the structural characteristics of an economy and the introduction of new and more productive technology.

To illustrate our proposition and for the purpose of keeping the presented model, as simple as possible, we limit ourselves from considering labor movements. Much of the existing growth theory takes the growth of technologies as sustained (Lucas and Moll, 2014) or random improvements. We assume that aside from those types of advancements, there also exists a long run growth pattern that affects one sector, which was first proposed by Nikolai Kondratieff (1935). In a recent study, Koroyatev and Tsirel (2010), used spectral analysis to confirm such pattern do exists. A clear example for that theory is the railroad expansion in USA during the second half of XIX century. Donaldson and Hornbeck (2016) estimated that, during the period of 1870-1890, the newly introduced railroads were critical for the agricultural sector, and removing them will lead to annual economic losses equal to 5.35% of GNP. A more modern example, for such technological wave, is the expansion of the IT and telecommunication sectors. The cyclical nature of technological progress is used to illustrate the dynamics of the FDI under the conditions of slow and fast growth.

The properties of these technological waves are described by Aghion et al. (2012)

and Helpman and Trajtenberg (1994): *first* when a new wave starts, the technology will grow very slowly, until a potential is reached; *second* the effects of these growth waves will affect other sectors; *third* they will enhance creation of new and better technologies. To capture these processes Helpman and Trajtenberg (1994) developed a endogenous growth model, based on Romer (1990), in which they called the technologies produced by the Kondratieff waves General Purpose Technologies (GPT) and defined them as innovations that are affecting the entire economic system. The presented approach replicates most of their results, by using exogenous R & D sector.

In order to explain, how this cyclical behavior is connected to foreign direct investments, we assume that the foreign investors will look for the sector with the highest rate of return and will try to earn as much profit, as possible. To maximize their rate of return, aside from physical capital, the investors will bring some knowledge and new technologies from aboard, in order to increase the output of the chosen sector. When the sector is only one, as assumed, the impact of these increases will affect production in two ways first by increasing the stock of physical capital, and second by introducing new technologies.

Our model also incorporates the country's structural characteristics, to include cases described by the Lucas Paradox. While this characteristics are different, all of them have one thing in common - they are system specific. For example in some of the countries in Central Africa, the corruption and wars are common, and no one will build company there, despite the low MPK and average wage. If we move to Asia - the entry barriers in Japan also partly prevents foreign companies from starting business there, despite the quality of labor and technological advancements. The other situation is also possible - when the structure of the economy allows FDI, and the investors are interested in investing, the capital will flow. A clear example for this is China - cheap labor force, almost no barriers for investing, clustered infrastructure and others. Changes in this characteristics will be considered exogenous, in order to show, how the economic system reacts to different structural policies.

In contrast with the most models, which are solved by searching for long run equilibrium, the presented model uses the balanced growth path only as an illustration for the Lucas Paradox .

# 2 The Model

### 2.1 General Assumptions

To describe our theoretical economic system, we start with the FDI. Each period there is an investor who invests some capital into the economy and acts as an additional capital lender. As previously assumed, FDI will depend on the structural characteristics of the economy: law system, market barriers, infrastructure, openness, corruption, political risk and so on. When the system is more attractive for the investment process i.e. lower systemic risk, better law system, better infrastructure, lower corruption, better public administration and new technologies the flow of FDI will increase. The rate of return for the investors will be connected, not only with the interest rate, but also with the profit their investment generate, so they will want to earn as much profit as possible, and will seek to invest only in a growing sector, which in the simplest case is one. If the sector grows fast, more investments will enter the economy from aboard, because the rate of return is high and vice verse. Of course there are always exceptions - some risk loving investor can find a way around the structural barriers of the economy, in order to earn higher profits. Using the same logic we can assume, that there will be some event that will be either positive or negative for the flow of FDI - a sudden investment, crisis, famine or war. The probability and the impact of such unexpected events are shown using a random variable.

The technological growth rate of a single sector exhibits a wave pattern with a single peak, as according to Kondratieff. When the cycle is near its end, a new sector is introduced, and the investors shift their investments from the old to the new sector. This assumption is again for simplicity - the technological progress in some sectors can be missing, but the sector will still produce output. If we continue our analysis, we can assume that Kondratieff waves can have multiple peaks, which will show that a sector can reach its maximum potential many times. In the same spirit a sector can "reemerge" after a certain period with zero technological advancements, because it depends on other sectors. This assumption leads to a model where the growth rate of technological progress in different sectors is partly correlated. An example for this is that the IT sector is connected with almost every other product - from washing machines and fridges to TV's and Air Conditioners. Aside from this cyclical behavior, the economy produces some new technologies and knowledge. In the endogenous

growth theory this type of growth is connected with the creation and distribution of knowledge, but here we assume it exogenous (for the basic framework), but endogenous for the economy. Again there exists some event that may speed up or slow down the growth of technology, such as new and unexpected inventions, shut downed projects and closed departments.

The interest rate in the country that receives the FDI and the interest rate outside of it are equal. That is the flow of FDI is not affected by interest rates and the international parity condition holds. Also there exists only one economy in which the investors can invest their capital, so they are not evaluating alternative investments. We also assume that exports equal imports and there is no government.

## 2.2 The Initial Setup

Time is discrete and there exists only one economy with only one sector and only one representative firm. The production function is Hicks neutral:

$$Y_t = A_t F(K_t, L_t)$$

Written in a Cobb-Douglas form:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

The production function satisfies the condition for constant returns to scale (CRS) and diminishing marginal productivity of capital and labor. The technological progress A evolves linearly:

$$A_t = (1 + g_g)A_{t-1} \tag{1.1}$$

Where g denotes the growth rate of technology, or the rate of innovation. In order to incorporate a cyclical pattern into the model we will assume that g can be decomposed into two different components:  $g_e$  - the endogenous (for the economy, exogenous for the model) growth rate and  $g_k$  - the growth rate accelerator. Also p stands for the probability for a random speed up or slowdown denoted by  $g_r$ :

$$g_t = g_e + ag_k + p(g_r) \qquad p \in [0, 1]$$
 (2)

The growth rate accelerator is a modified Gaussian function:

$$g_k = e^{-\frac{(\tau-b)^2}{2l}} \tag{3}$$

It creates a pattern movement in the trend of the growth rate g with length l, peak at period b, current period of the wave  $\tau$  and maximum growth rate a. Each time  $\tau$  reaches l, a new K-Wave with length l starts, and  $\tau$  assumes the value of 1. Substituting (3) into (2):

$$g_t = g_e + ae^{-\frac{(\tau-b)^2}{2l}} + p(g_r)$$
(4)

If we denote  $ae^{\frac{(\tau-b)^2}{2l}}$  as  $g_s$ , the current growth rate of the leading sector we can say that when  $\tau$  approaches b,  $g_s$  will approach its maximum a:

$$\lim_{\tau \to b} g_s = a \tag{4.1}$$

Because:

β

$$\lim_{\tau \to b} e^{-\frac{(\tau-b)^2}{2l}} = 1 \tag{4.2}$$

This form of innovation catches some important characteristics of the technological waves,- *first* the growth rate of technology during a new wave will start very slowly, because the technology is still new and needs time to develop. *Second* - after the peak is reached, a slowdown will eventually follow. Of course it is possible that certain technology stays at its peak for a long time. *Third* - there will exist a period when the old technology will were off or transform, so a new wave can start. In this initial setup there can only exist one technology without the possibility of branches or overlapping waves. In the basic framework the technology is produced outside of the economy and the absorption is not connected with the FDI or the structure of the economy.

In the initial setup the foreign direct investments are entering the economy as an additional source of capital:

$$I_{t} = \theta + \beta I_{q} + pS$$

$$\in [0, 1], \ p \in [0, 1], \ S \in [-\nu K_{t}, \nu K_{t}], \ \nu \in [0, 1]$$
(5)

Where  $\beta$  is the structural parameter of the model,  $\theta$  is the cyclical neutral FDI, p is the probability of a positive or negative FDI flow S with size  $\nu$  times the current

capital stock of the economy. The term  $I_q$  describes the choice based foreign direct investments, at time t, and can be written as:

$$I_q = g_k \beta_{t-1} q \tag{6}$$
$$q = \pi K_t$$

The term q denotes the investors available resources for an investment. As seen they take the lagged  $\beta$  into account, because there is lag in the information, how exactly the structural parameter affects FDI flows. Investors need one period to evaluate changes in  $\beta$ . We assume that households own the factors of production, and investors own additional capital which they are willing to lend at the same interest rate r. The zero profit condition for the sector also holds, indicating competitive market. The output is then distributed between the households and the foreign investors:

$$Y_{t} = w_{t}L_{t} + r_{t}(K_{t} - I_{q}) + r_{t}I_{q} + g_{k}\beta_{t-1}\pi Y$$

$$0 < \pi < 1$$
(7)

The terms  $-I_q$  and  $+I_q$  are here to show what is happening with the distribution of the capital costs - one part of the output is going to the domestic lenders and other part to the foreign ones. We denote the profit of the foreign investor with  $\pi Y$  where  $\pi$  represents the part of the income that goes to them. The profit of the investors will also vary, because it is connected with  $g_k$  and  $\beta$ . This is the reason for introducing  $g_k$  in eq.(6) - if the investors are not investing they will not reap any profit or receive interest. As assumed there is no government, net exports equal net imports and there is no difference between foreign and domestic interest rates -  $r^* = r$ . Under these conditions the disposable income equation will simply be:

$$Y_d = w_t L_t + r_t (K_t - I_q) = C + I$$
(8)

We know that c + s = 1 so the equation for the motion of the capital with the added FDI flow can be written as:

$$K_{t+1} = K_t + sY_d + I_x - \delta K_t \tag{9}$$

#### 2.2.1 Dynamics of the System

The dynamics of the model will be presented without taking into account random movements, because they will be separately described in section 2.4 and they do not change the results below. If we assume  $g_t=0$  and  $I_x = 0$  the system is reduced to a standard neoclassical growth model with a steady state condition at  $sY = \delta K$  (because  $sY - \delta K = 0$ ) and optimal capital stock  $K^*$ . When both equations (4) and (5) are introduced the model does not posses long run equilibrium, but instead we observe cyclical behavior generated by the Gaussian accelerator. The length of the cycles can be exogenous or stochastic. The second form has a particularly interesting property - it allows each new technology to posses a different lifespan. As observed foreign investors are not interested where to invest, but rather how much to invest. In the simplest version there is no other economy or sector, so how different countries and sectors are evaluated is not discussed here.

In order to choose how much to invest into the economy, the investors are looking at the structural parameter  $\beta$  and the growth rate of the current leading sector  $g_k$  jointly. So the necessary and sufficient condition for foreign direct investment flow can simply be written as :

$$0 < \beta \le 1 \quad , \quad 0 < g_k \le 1 \tag{10}$$

If the second of these inequalities is not satisfied -  $g_k = 0$ , there will be no investment activity and the economic system will converge to a balanced growth path as shown at Fig. 1(a). If the first inequality is not satisfied -  $\beta = 0$ , output will only grow thanks to  $g_t = g_e + g_k$ . If both conditions hold the system will exhibit cyclical patterns of technological growth and growth in investment activity. Both of these cases are presented at Figure 1(b). This is the main theoretical proposition of this paper - the only way an economy can attract FDI, is to raise its structural parameter  $\beta$  and in the same time there must exist a sector that will attract investors. As illustrated at Figure 1(b), we can observe that the closer  $\beta$  is to one, the more visible the technological cycles are. An interesting observation is that the attractiveness of this theoretical economic system is connected with the long run instability of the equilibrium. Despite this instability, because the economic system is absorbing technological cycles from aboard, the output at the bottom of each cycle is still higher than the output generated during a closed growth path. This leads to the following conclusion: the openness of this theoretical economic system contributes for two things : *first* non defined long run equilibrium and *second* growth. An international investment shock can decrease the capital for a short time, but in the long run the investment flow will continue. However the economic system can absorb such a shock by rising  $\theta$  in eq. (5).

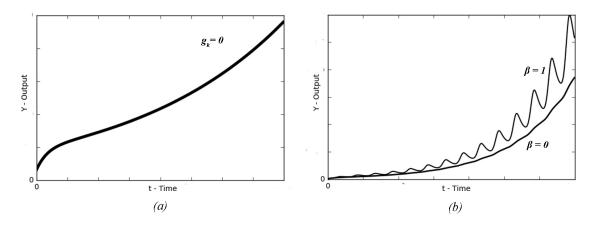


Figure 1: Dynamics of the output under different conditions

A note must be made here about this conclusions - we did not said anything about the size of q. In the previous paragraph, we assumed that the size of q is proportional to the rate of the profit  $\pi$  times the current capital  $K_t$ . This is the reason why the economy will not converge to a lower growth path - the higher rate of profit is linked with higher investment flows. The intuition behind this is that if the investor wants to receive higher payoff for his investment, he must invest more. This statement is oversimplified, because in this model the investors react only to the current situation, without considering the future. Also there is no consumption smoothing under a budget constraint, and there is no connection between risk and profits. So if we introduce a utility maximization approach, q will be subject to a budget constraint, which will include future interests and profits discounted to the current period. The problem with this is that the investors must optimize their utility flows under the assumption of an infinite time horizon, which implies that they are aware how long the technological cycles are and when their peaks will be reached.

Now what will happen if we assume that q is fixed, and it does not evolve under any conditions? If we invoke the property of the Cobb-Douglass production function for decreasing marginal productivity, the fixed value of q will depreciate after certain time, the cycles will again become barely visible. As shown at Fig.2 if we simulate the same economies side by side with  $q_1 = 10000$  and  $q_2 = 0$  after some time the growth rate of the both economic systems will converge. That's because even if we add a fixed amount of capital over time, the amount that depreciates will simply catch up.

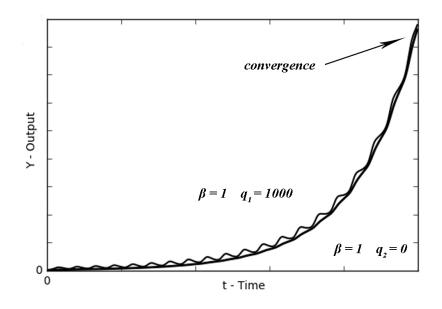


Figure 2: Convergence under a fixed q

This result leads to a very important conclusion - in the initial case the foreign direct investments must be constantly increasing in order to push the output up. If they stop at some time an identical economy, without FDI flows, will eventually catch up in terms of output growth.

This basic framework can now be extended to incorporate imperfection in international technology absorption.

## 2.3 Imperfections in Technology Absorption

In the previous section, we assumed a perfect absorption of technologies from aboard. This is not always the case, and can be easily illustrated by adding an absorption parameter in eq. (3) denoted by  $\mu$ :

$$g_k = \mu a e^{-\frac{(\tau-b)^2}{2l}} \tag{11}$$

$$0 \le \mu \le 1$$

The introduced parameter  $\mu$  is the same as  $\beta$ , but affects  $g_k$  instead of  $I_x$ . This way the choice function (6) is the same, because its already taking into account changes in  $g_k$ . Condition (10) can be rewritten as:

$$0 < \beta \le 1 \quad , \quad 0 < \mu \le 1 \tag{12}$$

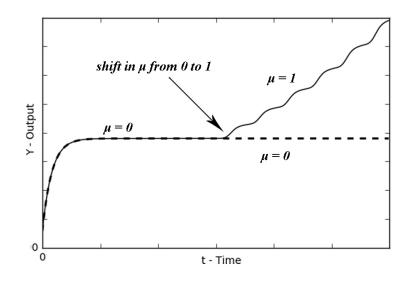


Figure 3: Shift in the technology absorption parameter with  $\beta = 0$ 

If  $\beta$  is zero the growth path of the output will be cyclical, but the cycles will be almost invisible in the trend, because they are not augmented by foreign direct investments. Changes in  $\mu$  will increase the output and the rate of growth as illustrated in Figure 3.

In this version the growth rate depends jointly on changes in  $\beta$  and  $\mu$ . Under the assumption that  $g_t = g_k$  (there is no endogenous or random technology growth),  $\mu = 0$  and  $\nu = 0$ , the growth path is reduced to a standard neoclassical steady state at  $sY = \delta K$ , even if  $0 < \beta \leq 1$  (Figure 4 (a)).

An imperfection in technology absorption leads to another question - how the foreign direct investments bring technology from aboard? One of the main empirical tests on the subject (Borensztein et al. 1998) showed that foreign direct investments affect output not only as an additional source of capital, but also as an additional source of technological growth. The basic illustration to this observation is straightforward.

Lets assume that the foreign investors have access to the cyclical technology, which is not available in the country that will be receiving FDI. They do not produce this technology, but can distribute it freely. This means that they are again taking into account the changes in  $g_k$ , because if  $g_t$  is zero (under the conditions of  $g_e = 0$  and  $\nu = 0$ ) there will be no growing sector in which the capital can be invested. The technology from aboard will be proportional to the amount of the FDI that the investors are investing into the economy - if they invest the maximum amount of their capital,

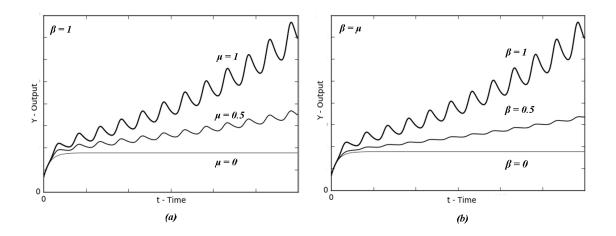


Figure 4: Dynamics of the system under imperfect technology absorption

they will also import as much as possible technologies. We can write  $\mu = \beta$ , because now they are jointly dependent. If  $\beta = 0$ , there will be no FDI flow, and no technology absorption. Because  $\nu = 0$  and  $g_t = g_k = 0$  the steady state condition  $sY = \delta K$  holds. The opposite is also true - if  $0 < \beta \leq 1$  the FDI will flow together with technology advancements (Figure 4 (b)).

The last described case is very interesting. The information about these new technologies is free, but their blueprints are not. Even today with all the free transfers of information, patent rights must be paid. This is the main problem of the modern growth theory. What  $\mu$  in the first case, and  $\beta$  in the second case do is to capture this property, which in the scope of the presented model can be used as a cause for the Lucas Paradox.

# 2.4 What about $\beta$ , $\mu$ and the random parameters?

In the beginning we said that  $\beta$  captures structural characteristics of the economic system. But how we can define what those structural characteristics are? We said that they include laws, levels of corruption, political risks, trade barriers and so on. What we must underline here is that  $\beta$  does not include factors of production. This is very important, because it means that investors do not take into account human capital or resource limits. Another thing that can be said about this structural parameters is that they include variables with qualitative nature. This limits the model, because one needs to modify the Cobb-Douglass production function in order to include difference in human capital between countries, which can lead to difference in FDI. Also the econometric transformation of the presented theoretical idea can be difficult to do, because there is hardly any available data for the exact level of lets say corruption in Third World countries.

There exists one more thing that is worth mentioning - the possibility of low  $\beta$  and high FDI. This is possible due to some way of exploiting the economic system, a clear example being investing under the protection of a Central African dictator, especially in gold and diamond mines. This means that small  $\beta$  can be misleading, because there may be a way of evading the system characteristics. That is way we included a random term in eq. (5). Of course the presented model is purely abstract and should not be used as an econometric tool in this form.

What about  $\mu$ ? The absorption parameter is even harder to describe in reality. It captures laws for international technology exchanges, the liberty of ideas sharing, the absorption of scientific discoveries, etc. But in the current digital world ideas and blueprints can be easily shared, which implies that some technologies may become available for production in a certain country without paying the necessary legal rights. This leads to forgery and imitation, but not under the market rules. Some Asian firms practice this type of imitation - they find a certain product and illegally start to produce it. Production of such a products does raise the technological advancement of the country, because the firm understands how the product works, but does not raise the FDI. No investor will want to invest in a sector, where firms wont be paying for the copyrights that he owns.

## 3 Conclusions And Policy Implications

With this paper we showed a model that combines multiple economic theories in order to explain the evolution of output and the Lucas Paradox in a long run. Investments are linked with leading sectors and structural parameters which is an interesting hypothesis, not yet discussed. Of course a test of this hypothesis must be conducted in the future. Technology also plays a key role under the conditions for imperfect absorption from aboard. However, the model presents some limitations, mainly the lack of utility maximizing agents and a better description of the economic dynamics. A future work may be conducted in this direction, combining the proposed framework with multiple agents, sectors and economies, overlapping K-Wave, learning based growth and creative destruction. In contrast with the literature, the presented model is focusing on structural characteristics and exogenous long run technological cycles to explain economic growth and the Lucas Paradox.

The policy implication of the model can be summarized as follows: in order to increase the flow of FDI and technological progress, the policymaker must increase the structural attractiveness for the foreign investor. Example for this is China's policy for foreign direct investments. Such government actions indeed attract international capital flows, but they are not enough. To fight the Lucas Paradox, one needs more structural reforms. However, there exists a certain limitation to that policy - while structural reforms can be very beneficial for the economy, it takes a long time for their effect to be seen. Meanwhile, in the short run, the policymaker must find a way to react to the international shocks that will hit the system. The combination between short rut stabilization and long run structural policies will create a favorable environment for the foreign investor. The policymaker should also take into account that while the FDI are generally beneficial for the economy, if they become too important the economic system will be more vulnerable to fluctuations in the international investment activity.

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