R E P O R T

on a Thesis for awarding the degree “Doctor”

Title: Algorithms for Characterization of Orthogonal Arrays

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Scientific field: 4. Natural sciences, mathematics and informatics

Professional field: 4.5. Mathematics

Overview

This thesis is devoted to problems from the theory of the orthogonal arrays. The latter are classical combinatorial objects with numerable applications in the statistics, coding theory, and cryptography. The goal of this thesis is to develop methods to settle the existence problem for orthogonal arrays with given parameters.

This thesis covers results from the author's research in the past 5-6 years.

Description of the results

This thesis amounts 115 pages of text and consists of an introduction, four chapters, and a list of references including 59 items.

Chapter 1 is introductory. The author gives the necessary definitions and main results needed in the thesis. In particular, section 1.1 introduces the Hamming space $H(n,q)$. In section 1.2 the author defines the orthogonal arrays and presents concisely some of their properties. In chapter 1.3 she gives some bounds om the parameters of orthogonal arrays obtained as different variants of the linear programming bound. In chapter 1.4 various spectra are defined; notably, the spectra of an orthogonal array with respect to an internal/external point of the array. various necessary conditions, i.e. equations satisfied by the spectra, are also given. They turn out to be the main tool in next two chapters.

The investigations in chapter 2 are focused on orthogonal arrays in the binary Hamming space. The main idea here is to use two classical constructions known from coding theory: shortening and puncturing of an array. Given an orthogonal array with parameters $(n, M, 2, \tau)$ the first of them gives an array with parameters $(n-1, M/2, 2, \tau)$, while the second yields a $(n-1, M, 2, \tau)$-array. The obtained arrays are called derived. In theorems 2.3.1, 2.3.3, 2.3.4, 2.3.6 the author presents restrictions on the spectra of the derived designs of a given

$(n, M, 2, \tau)$-array. The main idea is to violate all hypothetical spectra and by this to prove the nonexistence of an orthogonal array with given parameters.

In section 2.5 this line of thought is developed further by considering arrays obtained by shortening/ puncturing in two positions (columns).

The developed techniques are used further in sections 2.7 and 2.8 in order to prove the nonexistence of orthogonal arrays with the following parameters:

$$(20, 40, 3); (10, 96, 4), (11, 96, 4); (10, 192, 5), (11, 192, 5);$$
$$(12, 224, 5), (12, 112, 4), (11, 112, 4), (10, 112, 4), (9, 112, 4).$$

In Theorem 2.8.16 the author gives a list of 23 parameter sets for which there exists no orthogonal array.

In chapter 3 the author uses the approach from chapter 2 to orthogonl arrays in the Hamming space $H(n, 3)$. Results on the spectra of the shortened and the punctured arrays are obtained in section 3.2 (Theorems 3.2.7 and 3.2.10). They are used further to create an algorithm that finds the hypothetical spectra with respect to internal points of a ternary orthogonal array.

In section 3.3 the nonexistence of orthogonal arrays with parameters $(17, 108, 3)$ is proved.

In chapter 4 the author investigates energies of orthogonal arrays in the q-ary Hamming space. The main problem tackled here is to determine the minimal/maximal energy for which there exists an orthogonal array with parameters (n, M, q, τ). The main results here are Theorem 4.1.2 in which the energy of an orthogonal array is expressed in terms of all possible spectra with respect to internal points and their multiplicities, as well as Theorem 4.2.1 in which lower and upper bounds are found for different potential functions. In section 4.3 the author compares the various bounds on the energy of an orthogonal array. An algorithm is presented that simplifies and enables the computation of the energy of a given array.

In my opinion the main results of this thesis are the following:

(1) Results on the spectra of orthogonal arrays are proved related to internal and external points of the array.

(2) The spectra of orthogonal arrays are investigated for structures obtained by puncturing and shortening of a given orthogonal array.

(3) The nonexistence of orthogonal arrays with given parameters in the Hamming space $H(n, 2)$ is proved.

(4) The nonexistence of an orthogonal array with parameters $(17, 108; 3, 3)$ is proved. This implies that $\Lambda(17, 3, 3) \geq 5$.

(5) Lower and upper bounds on the energy of orthogonal arrays for different potential functions. The sharpness of the obtained bounds is investigated.

(6) Algorithms for easy applications of the developed techniques in this thesis are also presented.

Publications related to the thesis

The results of this thesis are published in six papers. Two of the papers are in journals with an impact factor:

- Problems of Information Transmission. (Q3, 2015, 0.632)

- Discrete Applied Mathematics (Q3, 2017,0.932)

From the remaining four papers one is in a journal with an SJR (Electrinic Notes in Discrete Mathematics), two are in refereed journals, and one is in the proceedings of an international conference. Three of the papers of Tanya Marinova are with three, two are with two, and one is with one co-author. I accept that her contribution is at at least as significant as that of the other authors.

The author provides a list of citations of the papers containing the results of the thesis. They show that that the results are well-known and positively evaluated by the professional community.

Remarks and comments

I have the following remarks, questions and comments related to this thesis:

(1) All upper bounds in Table 2.1 (resp. Table 3.1) excluded those in the first two columns (resp. the first column) are exact powers of 2 (resp. powers of 3). Is it true that all these constructions are linear, i.e. they come from linear codes with an appropriate dual distance? (Of course, we exclude the Nordstrom-Robinson code which is linear in a broader sense.)

(2) The orthogonal arrays C_0, C_1, C' obtained from a given orthogonal array C are called derived arrays. It will be more appropriate here to adopt the notions from coding theory and call the first two arrays shortened, and the third - punctured.

All the above remarks are insignificant mathematically and do not change my very good impression of the deep research carried out by the author.

Author's summary

The author's summary is made according to the regulations and reflects properly the main results and contributions of this thesis.

Conclusion

This thesis is focused on problems from the theory of the orthogonal arrays. The obtained results mark progress in this field of algebraic combinatorics.

I am deeply convinced that the presented thesis “Algorithms for Characterization of Orthogonal Arrays” by Tanya Todorova Marinova contains results that are an original contribution to the investigation of the orthogonal arrays. The candidate demonstrates deep knowledge of his field and the capacity to develop it in a new and important way. With this, she meets the national requirements prescribed by the law and the specific regulations of Sofia University and the Faculty of Mathematics of Sofia University for the professional field 4.5 Mathematics. I assess **positively** the presented thesis and recommend that this panel awards **Tanya Todorova Marinova** the educational and scientific degree “Doctor” in the scientific field ”4. Natural sciences, mathematics and informatics”, professional field ”4.5 Mathematics”.

Sofia, 19.04.2021 Member of the Scientific Panel:

(Prof. Ivan Landjev)