Position paper

by Prof. Ph.D. Krasimira Stoyanova Prodanova,

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of a Ph.D. thesis on:

COPULAS IN SOBOLEV SPACES AND APPLICATIONS

of Nikolay Kostov Chervenov,

presented for acquiring the educational and scientific degree

"DOCTOR"

Higher education: 4. Natural Sciences, Mathematics and Informatics.

Professional Field: 4.5. Mathematics,

Ph.D. study programme: Mathematical Modeling and Application of Mathematics in Economics

The presented dissertation (Ph.D. thesis) contains 98 pages. It consists of an introduction, three chapters and a conclusion outlining the main contributions of the dissertation, a list of six publications related to the dissertation, a list of reported results of conferences and a declaration of originality. The list of used literature contains 69 titles.

Ph.D.thesis meets the criteria and indicators acquisition the educational and scientific degree "DOCTOR" compliance with all laws and regulations in Republic of Bulgaria, Sofia University and Faculty of Mathematics and Informatics.

The presentation of the dissertation is clear and precise.

The work is in the field of creating probability models in more than one random variable, using copula functions, which allow constructing a joint probability distribution apparently through explicit formulas.

In contrast to the standard models that impose copulas on the data and examine whether the used copula (for example, Gaussian Copula, Clayton's Copula, T-Copula, etc.) is a good description of stochastic dependency (tests for feasibility), this Ph.D. thesis proposes a model in which a unique cupola is obtained as a solution to a problem with a differential equation.

In the first chapter "Bivariant copulas in Sobolev spaces", the case $n = 2$ is considered and a new general definition for the 2- increasing function is given. Equivalence with standard definition has been proved. A solution of constructing copulas based on a given

probability density is given using the weak derivatives (i.e. derivatives in the sense of distribution theory). In the presented approach, despite the constraints imposed by the use of Sobolev spaces, new copulas are obtained, useful in the processing of real data. The main result is the solution (in this case n = 2) of the boundary value problem under certain imposed conditions on the function *f* on the right side of the equation.

First, it is considered the case when f is a smooth function (paragraph 1.3), and then it is summarized for the more general case (paragraph 1.4). This problem can be thought of as a Dirichlet problem for the wave equation but in fact part of the conditions over the boundary are obtained by assumptions regarding the right hand side of the equation i.e. it only seems to be connected with the ill posed Dirichlet problem for the wave equation. The theorem for the uniqueness of the solution, and then its existence (theorems 1.4.1 and 2.3.1), has been proved under additional conditions for the right side of the equation.

The multidimensional case (n>2) for n-increasing functions is discussed in Chapter 2, where again two generalized definitions are given and equivalence with the standard one is proved. In this chapter, one important class, namely the class of Archimedes' Copulas has been introduced with details, and new proofs of Theorem 4.1.4 and Theorem 4.6.2 published in: Nelsen R. B., An introduction to copulas. Springer Series in Statistics, New York, Springer, 2006. The boundary value problem has been considered again and it has been proven that the obtained solution exists only under certain additional conditions.

Chapter 3 is devoted to the practical application of the developed method to evaluate the risk in insurance company. Real data from CASCO automobile insurance of a Bulgarian insurance company were used.

In literature, models describing the relationship between the magnitude of claims and their frequency most often use ready-made copulas, and the data are required to test whether the used copula is well-chosen.

The dissertation proposes a model that further examines the relationship between the magnitude of the claims, the time of their occurrence (relative to the date of the policy) and their frequency. For this purpose, a bivariate copula is constructed as a numerical solution of a boundary value problem. The copula is constructed in a way appropriate for the subsequent calculation of various measures of consistency and dependence.

From the constructed bivariant copula based on real data, the practical independence of the variable "insurance claim size" and "moment of occurrence of the event" has been established, since the coefficients of dependence are zeros with an accuracy of four characters.

The results presented in Chapter 1 are published in the Reports of BAS ([35]) and Serdika ([36]). The results presented in Chapter 2 are published in [5] and in the Reports of BAS ([67]) and Serdika ([68]) - in print. The results presented in Chapter Three are to be published ([69]). The publication numbers cited above correspond to those in the literature cited in the dissertation.

In brief, contributions to dissertation work can be grouped into four main groups:

• Two new generalisations of definition of n- increasing function and a proof of their equivalence with the standard definitions is given under certain conditions;

• New proofs of basic theorems for Archimedes Copulas using the introduced new definitions;

• Examining a mathematical model in which copulas are obtained as a solution of differential equation. Proofs of theorems for the existence and uniqueness of the solution in the two-dimensional and in the multidimensional case are given.

• Building a real-data model using data from insurance company that allows to investigate the relationship between the amount of claims, the time they occur (relative to the policy date) and their frequency. For this purpose a bivariant copula is considered as a numerical solution of a boundary task.

The dissertation of Nikolay Chervenov is a product of original research and represents a significant contribution to the knowledge in the subject area.

The author's summary of the dissertation reflects fully and exactly the content of the dissertation.

In view of the above, I strongly recommend that the Honorable Scientific Jury to award to Nikolay Kostov Chervenov the educational and scientific degree "Doctor" in the Higher Education area: 4. Natural Sciences, Mathematics and Informatics, Professional Field: 4.5. Mathematics, Ph.D. study programme: Mathematical modeling and application of mathematics in economics.

Member of the jury:

(Prof. Ph.D. K. Prodanova)

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