REVIEW

on a competition for an academic position

"Professor"

in professional field 4.5 Mathematics (Probabilities and Statistics),

for the needs of Sofia University "St. Kliment Ohridski" (SU),

Faculty of Mathematics and Informatics (FMI), Department of Probability, Operations Research and Statistics (PORS),

announced in the State Gasette issue 21 dated 13.03.2020 and on the websites of FMI and Sofia University (with extended deadline for submission of documents until 14.07.2020)

The review was prepared by: Prof. DSc. Maroussia Nikiforova Bojkova - Department of PORS at FMI-SU, in my capacity as a member of the scientific jury for the competition in the professional field 4.5 Mathematics (Probabilities and Statistics) according to Order № RD 38 - 267 / 10.07.2020 of the Rector of Sofia University.

The only candidate for participation in the announced competition who has submitted documents is **Assoc. Prof. DSc Mladen Svetoslavov Savov** from the Section "Operations Research, Probability and Statistics", IMI - BAS.

I. GENERAL DESCRIPTION OF THE MATERIALS PRESENTED

1. Details of the application

The candidate Assoc. Prof. Mladen Savov participates in the competition with all the necessary documents in accordance with the requirements of Act On Development of the Academic Staff in the Republic of Bulgaria (ADASRB), the Rules of Implementation of ADASRB (RIADASRB) and the Rules on the Conditions and Procedures for Acquiring Degrees and Occupation of Academic Positions at Sofia University (RCPADOAPSU).

For participation in the competition, the candidate Mladen Savov has submitted a list of a total of 33 titles of all publications, incl. 33 publications in foreign scientific journals, of which 14 articles were proposed for participation in the competition - 13 were published and one was accepted for publication. The articles proposed by the candidate are not submitted before for obtaining the educational and scientific degree PhD and the scientific degree "Doctor of Sciences", as well as for holding the academic position "Associate Professor"; and are not submitted to the National Center for Information and Documentation (NCID). There are also 11 other documents presented - official notes and certificates from the employer, participation in projects, financing organization or project assignor, awards and other relevant evidence supporting the applicant's high achievements.

The attached documents have been prepared in accordance with the requirements of the Law for development of the scientific staff and scientific degrees.

2. Professional and biographical data about the candidate

Mladen Savov graduated with a bachelor's degree in Mathematics at Sofia University "St. Cl. Ohridski" in 2004 with excellency for which he was awarded a prize by the "St. St. Cyril and Methodius" foundation. In the period 2005-2008 he was a PhD student at the University of Manchester in Great Britain, where under the supervision of Prof. Ron Doney he defended his doctoral dissertation on "Behavior of Levy processes around zero". As a doctoral student, he was awarded the "Doctoral Student of the Year" award by the Faculty of Engineering and Physical Sciences of the University of Manchester. From 2008 to 2009 he occupied a post-doc position at the Marie-Pierre Curie University in Paris, France, where he worked with Prof. Bertoin, and from 2009 to 2012 he worked as a research associate in mathematics at New College, Oxford. In 2011 he received the SCOPUS award for young scientist in mathematics. From 2012 to 2014 he was a lecturer in Probability and Statistics at the University of Reading, UK.

In 2014 he returned to Bulgaria and took a position of an Associate Professor at IMI - BAS in the scientific specialty "Probability Theory and Mathematical Statistics", where he currently works. He has also held a research position funded by the Maria-Sklodowska Curie Program, Horizon 2020 of the European Commission, as well as by the ACOMIN project at the Institute of Information and Communication Technologies of the Bulgarian Academy of Sciences. In 2017 he defended the scientific degree "Doctor of Science" at IMI - BAS with a dissertation entitled "Theory of exponential functionalities of Levy processes".

In 2014, the candidate Assoc. Prof. Savov participated in a project of the Bulgarian NSF at the Ministry of Education and Science. For his scientific activity Mladen Savov received financial support from British and Belgian institutions, respectively in 2009-2012 and 2011-2012. As an invited speaker, the candidate has participated in 20 international scientific events and over 20 seminars.

3. General characteristics of the scientific works and achievements of the candidate

To participate in the competition, the candidate submitted 14 scientific, 13 of which are published and visible in the world databases SCOPUS and / or Web of Science and the accepted paper is also in a journal with an impact factor. All the presented articles have not been used in previous procedures for obtaining the educational and scientific PhD degree and the scientific degree "Doctor of Science", as well as for acquiring the academic position "Associate Professor"; and have not been filed in NACID.

All 14 articles are in journals with an impact factor - a fact that is a clear evidence for the high quality of the candidate's research. The distribution by quartiles is as follows: 4 are in category Q1, 6 in Q2, 3 in Q3 and 1 in Q4 with a **total impact factor of 18,942**. Among the publications in which the candidate's articles are published are the highly esteemed ones in the field of the competition, such as: **Annals of Probability; Markov Processes and Related Fields; Bernoulli; Annals of the Henri Poincare Institute, Probability and Statistics; Mathematics and**

Computers in Simulation; Random Structures and Algorithms; Infinite Dimensional Analysis, Quantum Probability and Related Topics; Electronic Journal of Probability; Journal of Statistical Physics; Chaos, Solitons and Fractals; Electronic Communications in Probability. Below is a Table with the minimum number of points on indicators for the academic position "Professor" in the professional field 4.5 Mathematics at Sofia University and the indicators of Assoc. Prof. Savov, which shows that the candidate has excellent achievements and exceeds more than twice the required minimum.

According to the reference to the citations proposed in the competition, the total number of citations for the period 2017-2019 is 57, all in sources indexed in the world-famous database with scientific information SCOPUS and carry 456 points. The complete bibliography of citations includes 235 citations, of which 186 in peer-reviewed articles and 167 in sources with an impact factor.

Table. Minimum number of points according to indicators of the requirements under Art. 2b of the Law on the scientific domain "Natural Sciences, Mathematics and Informatics", professional field 4.5. Mathematics

Criteria	Requirements for „Professor" position	Indicators of Assoc. Prof. Savov
A. Dissertation work for PhD degree	50	50
B4. Habilitation work - scientific publications in journals that are referenced and indexed in world-famous databases of scientific information (Web of Science and SCOPUS)	100	516
G7 Scientific publications in journals that are referenced and indexed in world-famous databases with scientific information (Web of Science and SCOPUS), outside the habilitation work	200	315
D11 Citation in scientific journals, monographs, collective volumes and patents, referenced and indexed in world-famous databases of scientific information (Web of Science and SCOPUS)	100	456
E - total	100	160
Total	**550**	**1497**

Thus, the analysis shows that the scientific works of Assoc. Prof. Savov meet (under Art. 2b, para. 2 and 3 of ADASRB) and respectively the additional requirements of Sofia for acquiring the academic position "Professor" in the scientific field and professional direction of the competition. In my opinion, there is no legally proven plagiarism in the scientific papers submitted at the competition.

4. Characteristics and evaluation of the candidate's teaching activity

I have direct excellent impressions of the pedagogical work of Assoc. Prof. Savov. For the last three years, he has been teaching the subjects: Probability Theory 2 (compulsory course for Master's degree program); Random Processes (compulsory course for Bachelor's degree program) and Stochastic Processes 2 (compulsory subject for Master's degree program) at the PORS Department of FMI-SU. He has a serious and extremely responsible attitude towards students, combined with rigor and high requirements. At the same time, he is working to increase the interest in the field of "Probabilities and Statistics", in general, with great dedication, expressed in the organization of seminars, individual activities with students with strong interests in the same field and last but not least attracts them to the Master's program 'Probabilities, Actuarial Sciences and Statistics", as well as in the doctoral program "Probability Theory and Mathematical Statistics" at the PORS department of FMI-SU.

5. Analysis of the content of the scientific and scientific-applied achievements of the candidate, contained in the materials for participation in the competition

The scientific and scientific-applied activity of the candidate is in the field of the competition. More precisely, Assoc. Prof. Savov is **an internationally renowned expert** in the field of Levy processes and their applications and interplay with other known classes of random processes - Markovian, diffusion, Brownian motion and others, which requires appropriate knowledge of the properties and methods for their study in the same depth, as in Levy processes. He is applying properly methodologies from various fields such as: Krein's spectral theory of strings, differential equations, spectral expansions and asymptotic analysis, which is evidence of a broad-spectrum mathematical culture and abilities. Conditionally, the publications can be divided into the following areas:

(I). Classical properties of Levy processes, where publications with numbers [1, 11-13] are falling:

Article **[13]** is devoted to the so-called Chung's laws on the iterated logarithm (LIL) of Levy processes around zero and complements the topic of the dissertation for the PhD. More concretely, these laws study the minimum growth of the current maximum of a Levy process, finding a deterministic function that for small times gives a precise lower limit of the growth of the maximal process. This approach is new and makes it possible to construct a deterministic function for a large class of Levy processes only by means of the Levy measure. Compared to previous works by other authors, the advantage is that in this way the verification of Chung's law for a given process is performed with analytical quantities.

Article **[12]** proves for almost all Levy processes that the probability of a Levy process to remain t units of time in a finite interval decreases exponentially by t when t is increasing, with an exponential approximation error. The result is an improvement in Prof. Bertoin's work, both as a class of processes and as an accuracy in estimating the error of the approximation. The approach is new and consists in the application of the spectral theory of compact Markov semigroups in the field of Levy processes.

In the work **[11]** the classical question of the asymptotic of the probability of a Levy process to remain above / below an increasing / decreasing curve for a period of time, tending to infinity is investigated. For Levy processes falling within the domain of attraction of stable distributions, the family of curves for which it is possible to estimate this asymptotic has been extended. It turns out that on the logarithmic time scale it coincides with the one in which the curve is the constant unit. The new idea is to use a change of the measure for additive processes so that by a complex iterative procedure the problem for curve *f* is reduced to the constant limit / curve. Later on, a more powerful methodology of other authors was found and these results were significantly improved.

Article **[1]** gives a general solution to the question of whether it is possible to determine by an analytical criterion whether for a Levy process and a function *f*, it is true that $\int_0^\infty f(x+\xi_s)ds < \infty$ is satisfied almost surely. A universal, but often non-trivial, calculation criterion for the solution of this problem for all Levy transient processes and practically all measurable, nonnegative functions *f*, is proposed. The methods are new and based on a non-trivial generalization of the methodology developed by Batty for the Brownian motion.

(II). Diffusions, anomalous diffusions and stochastic processes with constraints, where publications with numbers [2,8,9,14] are falling:

The contributions in this direction are due to the inclusion of methodology in the field of diffusion processes in the theory of Levy processes. The field of anomalous diffusions is rapidly developing nowadays due to the fact that they meet the huge and diverse needs for modeling processes in which the typical diffusion behavior is not observed. In this regard, **Article [2]** discusses a class of such processes that generally describe the motion of particles in an environment with traps. The modelling of such a motion is done by $X_t = M(L(t))$, where *M* is the Markov process and *L (t)* is an increasing process with constant levels describing trap retention. Using the general theory of Markov processes, it is proved that the expectations $q_t = E[X_t]$ are solutions of general integral-differential equations with spatial dependence in their kernel. For the case where *M* is a Brownian motion and *L* is associated with fractional diffusion with a varying parameter reflecting the different strength of the barriers, the strength of the traps in a given region is investigated so that the process is likely to be observed there. Laws of repeated logarithms and Krein's spectral theory of strings are used essentially.

Article **[8]** is devoted to the Brownian motion/diffusion in a Poisson cloud of obstacles, which, independently of the diffusion, sets traps and when the motion gets into one of them, it is suspended. The behavior within the process boundary is studied with the condition that it does not meet a trap until *t*, when $t \to \infty$. In the case of Brownian motion with drift *h* and Poisson obstacle cloud (with intensity *v*) the behavior of the conditional process is well studied in dimension greater than 2 and in dimension 1. In paper [8] it is shown that in dimension 1 the behavior in the critical case, i.e. when $|h| = v$ is sub-ballistic (the boundary process increases diffusely), while in the literature it is known that in dimension over 1 the behavior is ballistic (increases linearly with

time). The methodology applied is specific for one-dimensional Brownian motion and uses spectral decompositions and asymptotic analysis.

Article **[9]** proves and improves the hypotheses of Prof. Benjamini and Prof. Berestiki regarding the limit behavior of one-dimensional Brownian motion with limited return to zero. The problem is as follows: if a non-decreasing deterministic function *f* is given, determine the limit behavior, if such exists, of the Brownian motion for which the local time at zero up to *t* is less than *f* and *t* converges to infinity. In [9] it was proved that the presence of a very long relative to others excursion beyond zero of the Brownian motion is leading in the behavior of the boundary process.

An article **[14]** discusses an industry problem described by the behavior of a specific stochastic differential equation that arises as a model of filter production when superimposing fibers on a surface by turbulent flow. The aim is to prove that the process is ergodic with a geometric rate of convergence. This is also the result of [14], as the main method consists in finding a Lyapunov's function for the specific equation.

(III). Spectral theory of Markov semigroups and applications of the exponential functionals of Levy's processes, where publications with numbers [5,7] are falling:

This is a field in which the candidate has been co-working in the last ten years with Prof. Patie (Cornell), and in which more than 7 publications have been jointly published. In fact, the theory of non-self-adjoin Markov semigroups, which are less studied due to the loss of symmetry, is considered and enriched. The main idea is to study the intertwining of non-self-adjoin semigroups with symmetric semigroups by a non-bijective operator. In particular, when studying semigroups related to self-similar processes, there is a need to develop a theory of exponential functionals of Levy's processes, the properties of which encode the basic characteristics of these semigroups.

Article **[7]** analyzes the characteristic properties of Markov semigroups that intertwine, i.e. $P_t\Lambda = \Lambda Q_t$. Specifically, if there is a regular point at which the Markov processes, X, Y associated with P_t, Q_t are killed and then continued in the Ito sense, then it is proved that the local times of X, Y in the regular point coincide in distribution.

In modeling different systems through random processes, their behavior up to a specific moment is often studied. For example, in population models, such a time is the time of extinction. There are many studies in the literature on moments of disappearance (reaching at level 0), when the stochastic process is Markov. In the article **[5]** for a class of self-similar stochastic processes, which are not Markov, complete information on Mellin's transformation of their extinction time is obtained, and from there many properties of the densities of the extinction times such as asymptotics, smoothness and so on, as well. The article uses the theory of exponential functionalities of Levy's processes, developed in co-authorship with Patie (Cornell). Thus, the extinction times for a rich class of processes can be studied in detail.

(IV). Some applications of branching and similar processes, where articles with numbers [3, 10] are falling:

In **[3]** the properties of the single particle method for the approximation of quantities related to the solution of the Wigner equation in quantum mechanics were studied. These methods are widely used in Monte Carlo simulations. The idea is to present the quantities of an equation as an infinite series, the terms of which are interpreted probabilistically as the expectation of the evolution of a particle, which undergoes a random change depending on the parameters and the character of the equation itself. In [3] there is an upper limit of the number of members of the infinite series that we have to approximate with Monte Carlo method so that the approximation is good. The applied technique is based on probabilistic estimates of the behavior of the stochastic process behind the Monte Carlo method and an appropriate embedding of the problem in a specific Hilbert space.

An article **[10]** discusses a general model of a birth and death population that takes into account the phenotype on which mutations, births, deaths, and competition between individuals depend. The process is presented as a point process in the space of phenotypes and is studied in a large initial number of individuals in the population. The equivalent of the strong law of large numbers for population evolution and the central limit theorem has been proved, which is a new result for such a general process. The disadvantage is that the proven similarity is in the space of Schwartz distributions, while the law of large numbers is in the space of measures.

(V). Contributions to financial mathematics and probabilistic combinatorics, where publications with numbers [4,6] are falling:

In **[4]**, the uniform probability measure on the set of all partitions of $[n] = \{1,2, \cdots, n\}$ is considered, i.e. all sorts of representations of [n] as a union of disjoint subsets. Various statistics are known in the literature related with this measure. In [4], a result of the type of central limit theorem (CLT) was proved, which shows the typical maximum multiplicity, i.e. the maximum number of subsets with same dimension in each representation of [n], as $n \to \infty$. It is important to note that the central limit theorem is not true on natural numbers, and it is valid on subsequences with specific properties.

The purpose of the article **[6]** is to present two different schemes for obtaining partial differential equations (PDE) for the price of the so-called default derivatives. In the first scheme, the asset price is set as a solution of a stochastic differential equation (SDE) stopped at random. The second one examines the effect of adding a jumping process, assuming that the stopping time is the moment of arrival of the first jump. The degree of asset loss in case of default was investigated. In both schemes, different assumptions and dependencies between the asset, the stopping time and the degree of loss are made and analyzed. The cases when the price of the asset is set in terms of the Brownian motion and with the Levy process are analyzed separately. A method for the decision of the PDE for the price of the derivative through the so-called default premium.

It is worth noting that all publications are co-authored with a total of at least 16 co-authors (according to SCOPUS), and here it is appropriate to note among them the names of world-famous mathematicians such as Bertoin, Doney, Kyprianou, Patie, etc., which is indisputable proof of the high quality of the candidate's research. I consider that his contribution to the joint publications is completely equivalent to that of the other co-authors.

6. Critical remarks and recommendations

I have no critical remarks.

7. Personal impressions of the candidate

I have known Assoc. Prof. Mladen Savov since his student years, when he attended the courses on Probabilities 2 and Stochastic Processes, which I led and made a strong impression on me as a curious and talented student with his in-depth knowledge and interests in the theory of probabilities and random processes. I follow with interest the development and its manifestations in recent years and I can definitely say that Assoc. Prof. Savov is growing as a world figure in the field of Probability Theory and Mathematical Statistics and in particular in the theory of Levy's processes and is distinguished by exceptional commitment and consistency in activities related to teaching and learning in the same field. It is important to add his leader abilities proved by the guidance of one PhD student from UK – Adam Barker - successfully defended his PhD degree in 2019.

Conclusion on the application

After getting acquainted with the materials and scientific works presented in the competition by the Assoc. Prof. Mladen Savov and on the ground of the analysis of their significance and the scientific and scientific-applied contributions contained in them, **I confirm that the scientific achievements of Assoc. Prof. Mladen Savov meet the requirements of ADASRB, the RIADASRB and the RCPADOAPSU for the occupation the academic position "Professor" in the scientific field and professional direction of the competition**. In particular, the candidate satisfies the minimum national requirements in the professional field and no plagiarism has been identified in the scientific papers submitted at the competition.

I give my **positive assessment of the candidacy**.

II. OVERALL CONCLUSION

Based on the above, I recommend the scientific jury to propose to the competent electoral body of the Faculty of Mathematics and Informatics at Sofia University „St. Kliment Ohridski" **to elect Assoc. Prof. DSc. Mladen Savov to take the academic position of "Professor" in the professional field 4.5. Mathematics (Probability and Statistics).**

5. 09. 2020. Reviewer: .....

(Prof. DrSci. Maroussia Bojkova)