R E V I E W

on a Dissertation

for obtaining the scientific degree "Doctor of Science"

Research area: 4. Natural Sciences, Mathematics and Informatics,

Professional field: 4.5 Mathematics

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Title: Finite Geometries and Codes

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I am a member of the scientific panel for this procedure according to order No. РД 38-186 / 14.05.2020 г. of the Rector of Sofia University ”St. Kliment Ohridski” Prof. D.Sc. Atanas Gerdzhikov. As a member of the scientific panel, I have received all the administrative and scientific documents required by the Act on the Development of the Academic Staff in the Republic of Bulgaria (ADASRB), the Rules for its implementation and the Rules on the terms and conditions for awarding of academic degrees and occupying of academic positions at Sofia University “St. Kliment Ohridski".

1. Personal data.

Assia Rousseva graduated in mathematics at Sofia University "St. Kliment Ohridski” in 1988. In 2005 she defended her doctoral dissertation titled “Arcs in Finite Projective Geometries and Their Application to Coding Theory” under the supervision of Prof. Ivan Landjev. Since 1993 she has been with the Department of Geometry at FMI, Sofia University “St. Kliment Ohridski ”, and since 2009 she has been an associate professor.

From 2013 she has been the head of a contract with the Science Fund of FMI, Sofia University “St. Kliment Ohridski” with the title “Finite geometries, incidence structures and applications". She has over 45 scientific publications, almost all in prominent international journals. She has participated in many international conferences in Bulgaria and abroad.

2. Relevance of the topic.

The dissertation is devoted to research on several problems in the area of Finite Geometries that are related also to the linear codes studied in Coding Theory.

There are several subjects of intensive research in Coding Theory. One of them is the construction and classification of linear codes with optimal and extremal parameters as well as proofs of bounds for these parameters. One of the main problems in Coding Theory is the investigation of the functions $n_q(k,d)$ (the minimum length n, for which an $[n,k,d]_q$ code exists), $k_q(n,d)$ (the maximum dimension k, for which an $[n,k,d]_q$ code exists) and $d_q(n,k)$ (the maximum d, for which there is an $[n,k,d]_q$ code). Most of the research presented in this dissertation is related to the function $n_q(k,d)$.

It turns out that for the investigation of codes over finite fields GF(q) for $q \geq 3$ it is very appropriate and successful to study their corresponding objects in finite geometries. The already mentioned basic problem for minimizing the function $n_q(k,d)$ from Coding Theory can be formulated naturally as a distribution problem for points in certain geometries over a finite field. This geometric interpretation is especially useful in theoretical research, constructions of codes, proofs for nonexistence or obtaining new bounds for the parameters. Effective research methods in this direction have been developed by leading scientists such as Belov, Logachev, Hamada, Hill, Helleseth, van Tilborg, Ball, etc. A detailed description of the representation of linear codes as multisets of points is presented in the work of Dodunekov and Simonis [56].

3. General characterization of the dissertation.

The dissertation consists of 180 pages and is structured in five chapters and references of 201 titles. The first chapter is introductory to the dissertation and gives a general idea of its content and structure. The second chapter presents basic information, definitions and important theorems from the theory of projective geometries and linear codes over finite fields. The next three chapters contain the original results of the dissertation.

Chapter 3 is devoted to geometric characterization of linear codes, whose length attends the Griesmer bound. These codes are called Griesmer codes. Because of this bound, many researchers have been working on the investigation of the function

$$t_q(k) = \max_{0\le d<\infty}(n_q(k,d) - g_q(k,d)), \qquad g_q(k,d) = \sum_{i=0}^{k-1}\left\lceil\frac{d}{q^i}\right\rceil.$$

This function gives the maximal deviation of the optimal length of a code of dimension k from the value given by the Griesmer bound:

$$n \ge g_q(k,d).$$

Our teacher acad. Stefan Dodunekov mentioned for the first time that for a fixed d and $k \to \infty$, the difference $n_q(k,d) - g_q(k,d)$ also tends to infinity, therefore $t_q(k) \to \infty$. The first three sections of this chapter examine the rate of increase of this function. The first section formulates three problems, which in practice represent the same problem, expressed in terms of linear codes (Problem A), arcs in finite projective geometry (Problem B), and blocking sets (minihypers) in PG(k-1,q) (Problem C). Several results simplifying the investigation of $t_q(k)$ are presented. One of the important assertions is Lemma 3.6, which shows that it is enough to compute the maximum of $t_q(k)$ only for a finite values of d:

$$t_q(k) = \max_{0\le d<q^{k-1}-q^{k-2}}(n_q(k,d) - g_q(k,d)).$$

The main result in Section 3.2 is Theorem 3.10, which can be viewed as a generalization of the construction of Belov, Logachev and Sandimirov for non-Griesmer codes.

The main result in Section 3.3 is a proof of Ball's hypothesis for planes of even order (Theorem 3.18). Section 3.4 presents the obtained exact values of the function $t_q(k)$ for q=4 and k=5. A characterization of the arcs with parameters (100,26), (117,30) and (118,30) in PG(3,4) is presented. This characterization is used to prove nonexistence of arcs with different parameters in PG(4,4). A table with all minimum distances d for which the exact value of $n_4(5,d)$ remains still undecided is given at the end of Chapter 3.

In Chapter 4, the extendibility problem for arcs in projective geometries and, equivalently, the associated with them linear codes is investigated. The dissertation proposes a new geometric approach to the extensibility problem, understood as the formulation of conditions under which the (n, w)-arc in PG (r, q) is extensible to (n + 1, w)-arc by increasing the multiplicity of one point. The main idea is to connect the extensibility of an arc with the structure of a special associated arc in the dual geometry. The extendability of the so-called arcs with t-quasidivisibility is of special interest. Such arcs are rather common for Griesmer codes with minimum distance $d \equiv -t \pmod{q}, t < q$. A general approach is used, in which the extensibility of an arc is associated with the construction of a specially defined arc in the dual space. Special objects are also introduced, called (t mod q)-arcs or arcs with superdivisibility, the characterization of which leads to the solution of important geometric problems. The developed techniques can be successfully applied to the main problem in Coding Theory considered in the dissertation.

Chapter 5 is devoted to constructions of blocking sets in the affine geometries AG(n,q). It consists of four sections. Section 5.1 is a survey on the known lower bounds for the cardinality of a blocking set in AG(n,q). The main result of this chapter is given in Section 5.2 – this is a new general construction of affine blocking sets (Theorem 5.6). Applications of this construction are described in Section 5.3. Section 5.4 presents two tables related to affine blocking sets.

The dissertation is very well balanced in content. The text is well formed, accompanied by professionally made drawings. Each chapter presents first the foundations on which the author steps in order to further develop the relevant theory and obtain new results. The literature of 201 titles shows a very good knowledge of research in the field of finite geometries and coding theory.

4. Contributions and importan e of the results obtained

I accept and approve the submitted contributions, indicated by Assoc. Prof. Rousseva. I would highlight the following contributions:

- Bounds of the function $t_q(k)$ for different parameters and different fields are proven:
    - For even dimensions the following bound holds true $t_q(k) \le q^{\frac{k}{2}}$.
    - For k=4 the following bound holds true $t_q(4) \le q-1$.
    - For even q the following bound holds true $t_q(3) \le \log_2 q - 1$.
    - For integers q, which are even powers of odd primes it is proven that $t_q(3) \le \sqrt{q} - 1$.
- 10 cases in the problem of determining the exact value of $\mathrm{n}_4(5,\mathrm{d})$ are solved.
- A new geometric object called a (t mod q)-arc is introduced. It is proved that the extendability of a t-quasidivisible arc is equivalent to the existence of a hyperplane in the support of special dual arc, which is a (t mod q)-arc.
- One of the four open cases for determining the exact value of $n_5(4,d)$ is solved.

- A general construction of affine blocking sets is proposed.

5. Publications and citations

The dissertation is based on 7 publications. All of them are written in English and published in international scientific journals. According to the national indicators for author-level academic metric in Bulgaria the points for publications on the dissertation are 177 (the minimum is 100 points) and 100 points from citations (minimum 100), so these indicators satisfy the minimum requirements of the law and the regulations.

Five of the publications are joint works with prof. Ivan Landjev, and the other two are independent works. My opinion is that Assia Rousseva's contribution to the joint publications is equal.

The research presented in this dissertation have been reported at many conferences on Finite Geometry, Combinatorics, Coding Theory and Cryptography in Bulgaria and abroad. Assia Rousseva presented a list of 13 citations.

6. The author's summary

The author's summary is made in accordance with the rules and accurately and completely assesses the main results obtained in the thesis.

7. Personal impressions

I have known Assia Rousseva for many years. I have listened to all her presentations at the ACCT (Algebraic and Combinatorial Coding Theory) and OCRT (Optimal Codes and Related Topics) conference series, as well as at the annual coding theory seminars. She is a very good lecturer, her talks are prepared accurately and precisely and presented convincingly. Assoc. Prof. Rousseva is a mathematician at a high level in the field of finite geometries.

8. Comments and Recommendations

My recommendation to Assoc. Prof. Rousseva does not refer to the content of her dissertation. I think it's a good idea to make her own accounts in the most popular networks for researchers like ReseachGate and Google Scholar. My experience shows that these networks promote the research of colleagues and allow better visibility of publications and citations.

The data on the publishing activity of Assoc. Prof. Rousseva on the FMI website are quite old and do not give a good orientation on her research. It would be good to update this information regularly.

9. Conclusion

The presented dissertation satisfies all the criteria and indicators of the law and the regulations in Bulgaria. After I familiarized myself with the presented dissertation, the importance of the research and the scientific and applied contributions contained therein, I give an overall positive assessment to the applicant **Assia Petrova Rousseva** to obtain the scientific degree "Doctor of Sciences" in

Research area: 4. Natural Sciences, Mathematics and Informatics,

Professional field: 4.5 Mathematics.

10.07.2020 г. Reviewer:

/Prof. Stefka Bouyuklieva/