

R E P O R T

by **Prof. Velichka Milousheva**

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
of the Dissertation Thesis
entitled

“The Geometry of Quaternionic-Contact Manifolds and the Yamabe Problem”

by

Assoc. Prof. Ivan Minchev Minchev

submitted for obtaining the scientific degree “Doctor of Science”
in the Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics,
Professional Field 4.5 Mathematics

I am a member of the Scientific Jury for the defense of this dissertation according to Order № RD 38-113 / 19.02.2020 of the Rector of the Sofia University “St. Kliment Ohridski”. The report is prepared in accordance with the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Sofia University “St. Kliment Ohridski”. From the required documents and papers submitted by Ivan Minchev, I have made sure that they and the applicant meet the requirements of Chapter 2 of the Act on Development of the Academic Staff in the Republic of Bulgaria. Ivan Minchev holds a PhD degree with a diploma No. 30962, issued on January 10, 2007 by the Higher Attestation Commission. The minimum national requirements for obtaining the scientific degree “Doctor of Science” in the professional field 4.5. Mathematics in accordance with the Regulations for the Application of the Act on Development of the Academic Staff in the Republic of Bulgaria are fulfilled.

1. General characteristics of the dissertation thesis

The dissertation thesis contains **208** pages and consists of an introduction, five chapters, a bibliography with **91** references and a content. The research is focused on current problems in modern differential geometry and mathematical physics, related to the theory of quaternionic-contact (QC) manifolds. One of the central points in the dissertation presented is the Yamabe problem on the quaternionic Heisenberg group. Investigating the relationship between the 3-Sasakian spaces and the QC geometry is one of the main tasks of the author. Essentially, the dissertation is theoretical in nature and meets the generally accepted requirements for obtaining the scientific degree “Doctor of Science”. The bibliography shows that Ivan Minchev has a good knowledge of the field in which he works.

2. Major scientific and scientific-applied achievements

In the first chapter, intended to be an introduction to the subject, the author explains the motivation for studying quaternionic-contact geometry (QC geometry) and recalls the main results known in this field.

In Chapter 2, the author develops the basic concepts and methods in the QC geometry and obtains some important results on which the rest of the dissertation is based. Theorems *A* and *B* give a partial solution to the QC Yamabe problem on the quaternionic Heisenberg group. Theorem *C* presents a result relating the Riemannian geometry of 3-Sasakian manifolds to the geometry of QC Einstein spaces. The results obtained in this chapter are published in the journal *Memoirs of the American Mathematical Society* (2014).

The research on the geometry of QC Einstein spaces continues in Chapter 3. Theorem *D* extends the result obtained in Theorem 5.9 of Chapter 2 and gives the answer to the most difficult 7-dimensional case: the author proves that the QC scalar curvature of a 7-dimensional QC Einstein manifold is a constant. Furthermore, it is shown that, depending on the value of the QC scalar curvature, the QC Einstein spaces are essentially bundles over quaternionic-Kaehler or hyper-Kaehler manifolds. The results obtained in this chapter are published in *Mathematical Research Letters* (2016).

In Chapter 4, a complete solution to the QC Yamabe problem on the 7-dimensional quaternionic Heisenberg group is obtained. The key results in this chapter are Theorem *E* and Theorem *F*. The results are published in *Journal of the European Mathematical Society* (2010).

In Chapter 5, the main focus is on determining the best (optimal) constant in the L^2 Folland-Stein inequality on the quaternionic Heisenberg group (in all dimensions) and the non-negative extremal functions. The main result is formulated in Theorem *G*. The results obtained in this chapter are published in *Annali della Scuola Normale Superiore di Pisa, Classe di Scienze* (2012).

The methods and results presented in this dissertation represent an original contribution to mathematical science and show that Ivan Minchev has in-depth knowledge in the field of the dissertation.

3. Approbation of the results

The results presented in the dissertation have been published in four papers in prestigious journals with high impact factor. Two of these papers were used for registration as an associate professor in the Register of Academic Staff at the National Centre for Information and Documentation. According to the information submitted by the applicant for the fulfilment of the minimum national requirements in accordance with the Regulations for the Application of the Act on Development of the Academic Staff in the Republic of Bulgaria in the professional field 4.5.

Mathematics, in the current procedure for obtaining the scientific degree "Doctor of Science" Ivan Minchev has applied with two papers with a high impact factor:

- Ivanov, S., Minchev, I., & Vassilev, D., *Quaternionic contact Einstein structures and the quaternionic contact Yamabe problem*, Mem. of AMS, Volume 231, Number 1086 (2014). IF: **1.727** (Q1)
- Ivanov, S., Minchev, I., & Vassilev, D., *Quaternionic contact Einstein manifolds*, Math Res. Lett., 23 (2016), no. 5, 1405-1432. IF: **0.716** (Q2)

Both papers are co-authored by Stefan Ivanov and Dimiter Vassilev. I accept that Ivan Minchev and all his co-authors have equal contribution to the joint papers.

The two papers mentioned above have a total of **29** citations according to the information provided by the applicant.

4. Critical notes and recommendations

The dissertation thesis in English is well written and designed. A number of technical and grammatical errors are noticed in the dissertation summary written in Bulgarian, which I will not dwell on, as they do not reduce the scientific value of the dissertation and do not spoil the overall good impression. In the English version of the dissertation summary, the year on the title page should be 2020, instead of 2019.

As a recommendation, I would suggest that it would be good to point out and list the main scientific achievements of the author at the end of the dissertation summary, as well as to indicate the perspectives for future work on the topic and for application of the achieved results in future research.

5. Qualities of the dissertation summary

The dissertation summary contains **42** pages and presents in detail the relevance and motivation for work on the selected topic, as well as the content of the dissertation by chapters.

6. Conclusion

The review of the proposed dissertation thesis, the summary, and the related scientific papers and documents shows that Ivan Minchev Minchev is an internationally recognized specialist in the field of differential geometry with significant personal contributions to the topic of the dissertation.

The above analysis shows that the submitted dissertation fully meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Sofia University "St. Kliment Ohridski". The scientific papers submitted by the applicant do not repeat those of previous procedures for acquiring a scientific degree and occupying an academic position. There is no data for plagiarism in the presented scientific papers.

All of the above gives me a good reason to give my **positive evaluation** of the dissertation thesis "**The Geometry of Quaternionic-Contact Manifolds and the Yamabe Problem**" and strongly recommend the Scientific Jury to award **Ivan Minchev Minchev** the scientific degree "**Doctor of Science**" in the area of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics.

May 7, 2020

Member of the Scientific Jury:

/Prof. Velichka Milousheva/