POSITION PAPER

by Prof. Dr. Georgi Penchev Venkov

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of a PhD thesis, entitled

COPULAS IN SOBOLEV SPACES AND APPLICATIONS

of Nikolay Kostov Chervenov

presented for obtaining the educational and scientific degree

DOCTOR

Higher education area: 4. Natural sciences, Mathematics and Informatics

Professional field: 4.5 Mathematics

Doctoral program: Mathematical modeling and application of Mathematics in Economics

I present this Position paper as a member of the scientific jury for awarding the educational and scientific degree Doctor to Nokolay Chervenov, a PhD student in the Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski" with scientific advisor Assoc. Prof. Dr Iordan Iordanov.

1. General description of the dissertation

The thesis is written on 100 pages and consists of Introduction, three Chapters and Bibliography, that refers to 69 scientific sources. The dissertation is based on five papers of Nikolay Chervenov, published in the period 2015 - 2019, as well as on one preprint (reference [69] in the Bibliography). The thesis fully complies with principles and requests for obtaining the educational and scientific degree Doctor according to Academic Staff Development in the Republic of Bulgaria Act (ASDRBA), the Regulation for the Application of the Act for the Development of the Academic Staff in the Republic of Bulgaria and all the Regulations of Sofia University and the Faculty of Mathematics and Informatics.

2. The topicality of the thesis

Copulas are extensively used in the fields of insurance, banking and finance and in the case of describing the dependence between interacting processes (risks). While the most common models impose copulas on the given data and study whether the copula (for example Gaussian copula, Clayton copula, *t* copula, etc.) describes correctly the stochastic correlation, the present dissertation

proposes a method for obtaining a specific copula as a solution to a differential equation (Goursat's problem) that corresponds to real data. The author considers copulas as functions in Sobolev spaces and aims to answer the following questions: how to determine whether a given function is 2-increasing and how to construct a new copula.

3. Aim of the scientific study

The main purpose of the present study is to construct copulas based on given probability density, using the concept of weak derivative (i.e. derivative in terms of distribution theory). Using this approach and despite some restrictions imposed by Sobolev spaces, are constructed new copulas, useful in the real data processing.

4. Important scientific contributions of the thesis

Here I shall list only the important contributions of the author in his study of copulas in Sobolev spaces.

- Two new generalizations of the notion of an n-increasing function are given and it is proved their equivalence under certain conditions. In the case of smooth functions it is shown that they coincide with the classical definition of n-increasing function. The applicability of the new notions is verified with many examples and it is practically proved that they lead to simplification of the determining whether a function is n-increasing;
- For the class of Archimedean copulas, using the new notions for n-increasing function, are proved the main theorems for determining whether a function, constructed as an Archimedean copula is n-increasing;
- It is defined the Goursat's problem over the n-dimensional unit cube for copula belonging to a specific Sobolev space and for a given probability density. The corresponding existence and uniqueness theorems are proved for n=2 and then the proof is generalized for arbitrary n>2;
- Based on the new method, developed in the thesis, it is constructed a copula as a solution to a concrete Goursat's problem with actual data taken from one Bulgarian insurance company. The solution of this equation is obtained by using a spectral numerical method.

5. Publications related to the thesis

The thesis autoreferate and the dissertation list 5 published articles and one preprint of the author. All of them have one or two co-authors and are published in the period 2015-2019. The papers are published in scientific journals and conference proceeding volumes such as: Comptes rendus de l'Academie bulgare des Sciences, Serdica Math. J. and AIP Conference Proceedings, while three of them have Impact factor (IF) or SCImago journal rank (SJR). Part of the results are presented at different scientific forums like Spring conference of the Faculty of Mathematics and Informatics and the International conference "Applications of Mathematics in Engineering and Economics".

6. Thesis autoreferate

The thesis autoreferate of the dissertation "Copulas in Sobolev spaces and Applications" reflects fully and exactly the content of the dissertation.

7. Critical remarks

I don't have any critical remarks or comments on the dissertation, the thesis autoreferate and the additional documents of Nikolay Chervenov.

CONCLUSION

The dissertation "Copulas in Sobolev spaces and Applications" presents the results of a meaningful and in-depth research in the area of copulas. I can give a positive evaluation of the thesis and think that it fully complies with the requests for obtaining the educational and scientific degree Doctor according to Academic Staff Development in the Republic of Bulgaria Act. In view of the above, I recommend the Honorable Scientific jury to award to Nikolay Kostov Chervenov the educational and scientific degree "Doctor" in the Higher education area: 4. Natural sciences, Mathematics and Informatics, Professional field: 4.5 Mathematics, Doctoral program: Mathematical modeling and application of Mathematics in Economics.

28.03.2019

Member of SJ: Prof. Dr. Georgi Venkov