Report about the thesis submitted by Tatyana Ivanova to Sofia University

Title

Logics for Relational Geometric Structures: Distributive Mereotopology, Extended Contact Algebras and Related Quantifier-Free Logics.

Referee

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Subject

The thesis of Tatyana Ivanova falls within the framework of the research dedicated to developing Whitehead's ideas about point-free theories of space. Whitehead's ideas have been recently rediscovered and developed in areas like knowledge representation and qualitative reasoning. It is a topical subject that has many possible applications in computer science and computational linguistics (geographical information systems, natural language understanding, etc) and where there are many mathematical questions still to be answered (expressive power, axiomatization, completeness, decidability, etc). In this setting, following the line of reasoning suggested by [Randell *et al.* (1992)] and furthered by [Wolter and Zakharyaschev (2000)], one usually considers, in a given topological space, the Boolean algebra of its regular closed subsets together with mereotopological relations such as external-contact, overlap, tangential-part-of, non-tangential-part-of, etc. Given a first-order language interpreted over these mereotopological relations, the main question is the decidability and complexity of the associated satisfiability problem. See [Kontchakov *et al.* (2013)] for a recent paper on that subject. Another line of research that has a bearing on Whitehead's ideas attempts to establish correspondences between point-free mereotopological models such as Boolean algebras with contact relations and point-based topological spaces. The main task, here, is to determine necessary and sufficient properties of the relations that constitute these mereotopological models in such a way that we can equivalently represent them as binary relations defined in topological spaces. See [Düntsch and Winter (2005)] and [Dimov and Vakarelov (2006)] for important papers on that subject.

Tatyana Ivanova aims to contribute to the definition of relational systems consisting of regions together with mereotopological relations such as *part-of*, *contact*, *non-tangential part-of* and *dual contact*. The binary relations of non-tangential part-of and dual contact are definable from the binary relation of contact by means of the complementation operation. Hence, a question arises: how to axiomatize the theory of these binary relations in a complementation-free language? In this setting, Tatyana Ivanova has taken up the following ambitious challenges: (i) axiomatization of the theory of extended distributive contact lattices (EDCL), i.e. the theory of the binary relations of part-of, contact, non-tangential part-of and dual contact in a complementation-free language; (ii) relational representation theorems stating that EDCL are embeddable into contact algebras; (iii) topological representation theorems yielding representations of EDCL into regular closed or open subsets of topological spaces; (iv) extensive study of the unary predicate c° of internal connectedness and the ternary predicate $\vdash$ related to the intersection operation and the relation of part-of; (v) axiomatizations and representation theorems for the extended contact algebras (ECA) based on c° and $\vdash$. To successfully achieve her aim, Tatyana Ivanova proposes an approach based on classical tools and techniques such as representation theory by means of filters and ideals, p-morphism lemma, etc. Her appropriate use of these tools and techniques allows her to prove difficult results such as representation theorems, complete axiomatizations of the quantifier-free fragments of the first-order theories of mereotopologies, decidability of some of these fragments.

Structure

The thesis of Tatyana Ivanova is 71 pages long. It comes with an abstract of 17 pages that methodically and truthfully describes the principal results of the thesis. The thesis itself consists of 4 chapters, the last one being a conclusion (Chapter 4). The list of the publications of Tatyana Ivanova that are related to the thesis is given at the end of her 17-pages abstract. It comprises a publication in a conference — *Panhellenic Logic Symposium* — and a publication in a journal — *Annals of Mathematics and Artificial Intelligence*.

Content

In Chapter 1, Tatyana Ivanova extends the complementation-free language of distributive contact lattices by means of the binary relations of non-tangential part-of and dual contact. She obtains an axiomatization of the quantifier-free fragments of the first-order theories of EDCL in a complementation-free language and she proves relational representation theorems stating that EDCL are embeddable into contact algebras. She also obtains topological representation theorems yielding representations of EDCL into regular closed or open subsets of topological spaces. Her results have been obtained by means of tools and techniques based on filters and ideals, separation lemmas for filters and ideals and extension lemmas for filters and ideals.

In Chapter 2, Tatyana Ivanova considers the new predicates c° of internal connectedness and $\vdash$ related to the intersection operation and the relation of part-of. She proves definability and non-definability results about these predicates, showing that point-free languages based on them are more expressive than ordinary languages based on the binary relation of contact. Defining ECA as contact algebras with c° and $\vdash$, she gives an axiomatization of the quantifier-free fragments of the first-order theories of ECA and she proves topological representation theorems stating that ECA are embeddable into topological contact algebras. Her results have been obtained by means of a generalization of the notion of ultrafilters, the notion of abstract points.

Chapter 3 is about several logics based on a quantifier-free first-order language interpreted over EDCL and ECA. Some of these logics are axiomatically presented by means of special rules of inference. Tatyana Ivanova compares together these logics and examines their completeness with respect to such or such classes of EDCL. She also studies their computability.

Chapter 4 consists of a conclusion summing up the main results of the thesis.

Evaluation

The thesis of Tatyana Ivanova is original and greatly contributes to the development of Whitehead's ideas about point-free theories of space. Her important results — axiomatization of the theory of EDCL, relational representation theorems stating that EDCL are embeddable into contact algebras, topological representation theorems yielding representations of EDCL into regular closed or open subsets of topological spaces, extensive study of the predicates c° and $\vdash$, axiomatizations and representation theorems for ECA — have been obtained by an approach based on classical tools and techniques such as representation theory by means of filters and ideals, p-morphism lemma, etc. Tatyana Ivanova skilfully uses these classical tools and techniques and shows her strong capacity in rigorously proving difficult mathematical results. For instance, the proof, in the first part of Chapter 1, of the embedding of EDCL into contact algebras is 8 pages long and necessitates no less than 8 intermediate lemmas. Another example, in the second part of Chapter 1, the proof of the topological representations of EDCL necessitates the long and tedious verification of multifarious cases. In order to prove so many intermediate lemmas or to verify so many cases, one has, first, to have a thorough knowledge of the different tools and techniques used in this thesis and, second, to possess the mathematical skills that are required to rigorously apply them. And Tatyana Ivanova, certainly, has this thorough knowledge and certainly possesses these mathematical skills.

I have been able to read and check the validity of the arguments displayed in the proofs of the thesis' results. Remind that some of them necessitate the proof of many intermediate lemmas or the verification of many cases and Tatyana Ivanova succeeds in convincing the reader of their validity, especially when she is embedding EDCL into

contact algebras and when she is proving the topological representations of EDCL. Nevertheless, she is less convincing when he is studying, in Section 1 of Chapter 3, the properties of L-theories — L being defined by means of some infinitary rules of inference at Page 49 — and the properties of structures determined by maximal L-theories. For instance, it is not clear, in the proof of Proposition 1.1 at Page 53, what kind of rules of inference is considered. Another example, in the proof of Proposition 1.2 at Page 53, where do the terms $a_1, \ldots, a_n$ and $b_{i_1}, \ldots, b_{i_{m_i}}$ come from? I believe that an appropriate rewriting of this section will allow the reader to better evaluate the contributions of Tatyana Ivanova they encompass.

Conclusion

Apart from the minor defects discussed above, I consider that the thesis of Tatyana Ivanova contains a lot of interesting and new results in mathematics (Mathematical Logic) and that their proofs — in spite of their high complexity and unusual length — have been rigourously presented. For all the above-mentioned reasons, I consider that Tatyana Ivanova deserves to receive the title of doctor in mathematics (Mathematical Logic) of Sofia University.