REVIEW

by Professor Nikolay M. Yanev, Dr. Sci., Institute of Mathematics and Informatics, BAS, member of a scientific jury in a promotion (competition) for the title of the academic position of "Professor" at FMI-SU according to PN 4.5. Mathematics (Probabilities and Statistics)

I. Requirements to the candidate

The promotion refers to the academic position of "professor" in a professional field 4.5. Mathematics (Probabilities and Statistics), announced in the State Bulletin no. 21 of 13.03.2020 for the needs of FMI at Sofia University. The deadline for submission and acceptance of documents under the announced promotion has been extended until 14.07.2020. The jury was appointed by order of the Rector of Sofia University "K1. Ohridski" RD 38-267 / 10.07.2020.

The only candidate in the promotion is Associate Professor **Mladen Svetoslavov Savov**, Dr. Sci., who currently holds this position at IMI-BAS, Section "Operations Research, Probability and Statistics (IOPS)". The submitted documents for participation in the promotion were reviewed by a special commission according to the order of the Rector of Sofia University RD 38-267 / 10.07.2020, which includes the Dean of FMI Assoc. Prof. Dr. Parvan Parvanov, Head of the PIOS Department Assoc.Prof. Dr. Vesela Stoimenova and Prof. Dr.Sci. Maroussia Bojkova. The Commission has drawn up a protocol stating that the documents are regular, in full and the tender procedure can continue.

For evaluation in the promotion (competition) I received 20 documents, which are described in detail in the application for participation in the competition. Here are some of the more important ones: CV, diplomas, list of publications and copies of them on technical media, reference for contributions and citations, reference for fulfilment of the minimum national requirements and the additional requirements under ZRAS, etc., all duly prepared, which gives me grounds to accept them for consideration and to establish that all formal requirements of the procedure are met.

The candidate's CV is prepared in accordance with the requirements of the European model and contains data on a number of serious achievements. Here are some of them.

During the period 2000-2004 M. Savov was a student at FMI-Sofia University, where he completed the bachelor's program with excellent results and the award "St.St. Cyril and Methodius" of Sofia University. In 2005-2008 he was a graduate student at the University of Manchester, UK, under the

2

supervision of R. Doney, where he defended his dissertation on "Small time behaviour of Levy processes" and received the title of "Doctor". It should be noted that during this period he had a special doctoral scholarship, and in 2007 he received the "doctoral student of the year" award. In 2008-2009 he was a postdoctoral fellow with Prof. Bertoin at the Pierre and Marie Curie University, Paris. From 2009-2012 he was a Research Fellow in Oxford, and in 2012-2014 he was a lecturer in Probability and Statistics at the University of Reading, UK. In 2011 he received the Scopus Award for Young Scientist in Mathematics in the UK. In 2014 he received the title of associate professor at IMI-BAS, where in 2017 he defended his dissertation for "Doctor of Science" on "Theory of exponential functionalities in Levy's processes" in PN "Mathematics", specialty "Probability and Statistics". During the period 2011-2017 he received 3 grants and participated in four projects, one of which is to the National Research Fund (2014), and the other three are international (two individual and one joint). He speaks English, Spanish and French, as well as three programming languages.

M. Savov's scientific interests are generally in the field of probability theory and stochastic processes. Among them, special attention is paid to various aspects of Levy and Markov processes, as well as to mathematical modelling of particle motion and Monte Carlo methods. During the period 2008-2020 M. Savov worked on 33 articles, of which 30 were published and three were accepted for publication. All articles are in renowned international journals with an impact factor. There are also two articles for print. All works are in English. There are 23 talks presented at international conferences (3 of which are in Bulgaria), 20 talks were presented at seminars, 3 of which were at the National Seminar on Stochastics and 17 at a number of Western universities.

Of all the publications for participation in the competition, 13 printed articles and one accepted for publication were presented, and all these works did not participate in previous competitions and defences, respectively for "Doctor", "Associate Professor" and "Doctor of Science". These works will be analysed in more detail in the next part of the review: 4 of them are in Q1, 6 in Q2, 3 in Q3, 1 in Q4. Nine of the articles carry 516 points on group indicator B, and the remaining 5 are on group indicator D and carry 315 points, in accordance with the Rules of the IAS in Bulgaria. There are 57 citations that carry 456 points. It should also be noted that M. Savov is a research supervisor of a successfully defended doctoral student in England. In general, the attached reference to the minimum scientific requirements shows that they are undoubtedly exceeded.

In this regard, we must note the excellent teaching activities of M. Savov, which fully covers the hours of a professor. Thus, in 2012-2014 he taught courses in Probability and Statistics at the University of Reading, UK, and in 2017-2020 he lectured on Probability Theory 2 and Stochastic Processes at FMI - Sofia University.

A detailed report (in 6 pages) on the contributions of scientific papers and a separate 37 pages of summaries of scientific publications are presented. A

reference to 57 citations indexed in known Scopus databases is also attached. These documents will be discussed in detail in the next section of the review.

Finally, it should be noted that currently M. Savov is Chairman of the Bulgarian Statistical Society, a member of the Scientific Assembly of IMI - BAS and a member of the editorial board of Serdica.

From the inspection there is no finding for absence or presence of violations in the procedure and for inadmissibility of the candidate to the competition. On the contrary, the high quality of the submitted materials for the competition and the full satisfaction of all formal regulatory requirements are obvious. All this gives me a reason to proceed to the next sections, according to the relevant regulations.

II. Research analysis and scientific and applied activity

First of all, we should note that the 14 articles presented in the competition (one of which is in print) have been published in a number of renowned international journals with an impact factor. Undoubtedly, two publications in "Annals of Probability" - one of the leaders of world Stochastics, as well as two works in "Bernoulli" make a strong impression. There is one article each in the well-known stochastic journals: "Markov processes and related fields", "Random structures and algorithms", "Annals of the Henri Poincare Institute", "Electronic J. of Probability", "Electronic Communications in Probability", "Infinity dimensional analysis, quantum probability and related fields". In addition, one article has been published in physical journals: "J. of Statistical Physics", "Chaos, Solitons and Fractals". One paper was published in "Mathematics and Computers in Simulation" and another in "Siam J. of Mathematical Analysis". Eight of the articles are co-authored with one co-author and six with two co-authors. Of all these co-authors, 4 are our colleagues from Bulgaria, and the other 10 are from abroad. The leading role or at least the equal co-authorship of M. Savov is not in doubt.

In the attached scientific report, the candidate has divided the submitted articles into five groups (2 by 4 articles and 3 by 2 articles). They briefly give the main directions of research and explain some of the main results. Reading only the reference and summaries, one can be left with the impression that there are no proven theorems, as such are not cited. And this is not the case at all.

For example, the main result of [1] is presented in Theorem 2.1, which gives a necessary and sufficient condition for the divergence of an integral of a sufficiently wide class of functions by a potential measure of a class of Levy processes converging to infinity. This in turn leads to an almost certain convergence of the stochastic integral of the corresponding function with an argument of the Levy process. Note that this work was published in "Bernoulli" in 2020 and is from the first group of articles [1, 11-13], which discusses generally speaking "classical" properties of Levy's processes. In this respect, the main results in [11] are interesting, which are presented in the respective Theorems 1 and 2, related to the so-called negative and positive limits for Levy processes. Generally speaking, Levy's processes in the field of attraction of stable distributions are considered, for which a class of "barrier" functions is determined, which retain this property. The article [12] investigates the asymptotic behaviour of the probability of the Levy process staying in a finite interval (0, a), a > 0, when time tends to infinity. The main result is presented in Theorem 2.1, where some questions related to the spectral theory of Markov semigroups are investigated. Interesting is the basic inequality (2.7) and the next Yaglom limit (2.8), i.e. quasi-stationary conditional limiting distribution. A similar result was proved by Yaglom for subcritical branching processes. Theorem 2.1 uses two conditions (F) and the weaker one (DF), as stated in the paper. The article [13] examines the issue of so-called LIL (law of iterated logarithm) on the asymptotic behaviour of Levy processes around zero. Let us recall that the LIL was proved first in the foundational works of Hinchin (1924) and then of Kolmogorov (1929) on sums of independent and identically distributed random variables. Now in Theorem 2.1 and Theorem 2.2 generalizations of these laws are given in a sense. In addition, a number of interesting consequences of them are indicated. In general, the work of this first group can be said to have used a variety of original techniques, which leads to new and interesting results on different classes of Levy processes.

The second group of articles [2, 8, 9, 14] deals with some more non-standard problems related to different classes of stochastic processes. Thus, in [2] we start from an integro-differential equation (1.3) of Volterra with spatially inhomogeneous kernels, and in the right part there is a generator of a Markov process. It is shown that in this way a suitable class of semi-Markov processes can be compared. The article is published in "Annals of L'Institut Henri Poincare" and covers 38 pages. The main results are presented in several statements and theorems. Thus, Theorem 3.7 presents the solution of equation (1.3) as a mathematical expectation. Note also that the situation is quite complicated by the presence of so-called "traps". The case when the leading process is one-dimensional Brownian motion is also considered. The main results are presented in Theorem 4.1, Theorem 4.12, Theorem 4.13 and Theorem 4.17. The numerous consequences are also interesting. After all, these processes with the so-called anomalous diffusions can be interpreted as models of particle motion in porous media. The article [8] investigates a Brownian motion process with drift h, up to the moment of sanding T with a Poisson process with intensity μ . The so-called critical case in which $|h| = \mu$ and the obtained main results are presented in Theorems 2.3, 2.4 and 2.8. Interesting "ballistic" properties related to the dimensionality of Brownian motion have been found. The article [9] was published in "Annals of Probability" (2016) and investigates an interesting problem related to the Brownian motion process provided that the local time at zero is limited by a growing function (deterministic) to some point T. When T tends to infinity a class of limiting functions is found, so that there is a conditional boundary process, for which, in turn, necessary and sufficient conditions for transitivity are found. The main results are presented in five theorems, which in particular generalize the results of other authors. There is an interesting phenomenon, "single big jump". By the way, a similar phenomenon was discovered by Feller for a simple symmetrical random walk. In the last article of this group [14] the superimposition (twisting) of fibers in turbulent air flow is modelled, which results in a filter. The model is described by a system of two stochastic equations, where "stochasticity" results from a Brownian motion process associated with turbulence. Three theorems have been proved that describe different properties of the system and ergodicity (with geometric velocity) has been proved.

The third group is based on articles [5, 7], in which problems related to exponential Levy functionals and spectral theory of Markov semigroups are considered. Note first that work [5] was published in "J. of Statistical Physics (2019) and considers interesting problems regarding the moment of death T (degeneration, annihilation, disappearance and other synonyms) for a class of self-similar Markov processes. In this case T is the moment of first reaching a negative level and Melin's transformation is studied. The main results are formulated in Theorem 1.1 and Theorem 1.7. Exponential functionals of the Levy and Bernstein-Gamma functions are used. Interesting examples are also given. The article [5] was published in the "Annals of Probability" (2019) and covers 46 pages. Several interesting problems related to Markov semigroups are considered. For two Markov processes X and Y, the so-called "killed" (stopped) semigroups of transient operators P and Q to some boundary point b, and then continue in the sense of Ito. A number of properties related to the so-called entanglement. The main results are presented in three theorems. There are interesting implications, applications and interpretations.

The fourth group includes the articles [3,10], in which, generally speaking, population models are considered. Note first that in [3] the Wigner equation of quantum mechanics is studied. After well-known transformations, it is reduced to an operator equation, for which the uniqueness in Theorem 1 is proved in particular. Two more theorems are proved. The main research method is related to Monte Carlo simulations and interpretations about the random evolution of "particles". Often these are different classes of branching processes. The work is jointly with I. Dimov and is published in "Mathematics and computers in simulations". The article [10] generally examines the behaviour of a number of population processes of death and birth with a finite number of types of individuals. The interesting thing here is the dependence on time, both on the number of types and the type of these types. The processes being studied actually represent the total number of individuals at a given time (there is an annoying technical error in the definition). When the initial total number of individuals v 0 grows linearly by n, it is a known law of large numbers, that is

 v_t / n tends to X (t) for n tends to infinity. Under certain conditions for v_t / n , an analogue of CLT (central limit theorem) is proved in the work.

The last fifth group of articles [4,6] deals with problems related to combinatorial probabilities and financial mathematics. Thus, in [4] an interesting combinatorial problem is considered, related to decompositions of integers and the uniform measure on them. The work is joint with L. Mutafchiev and is published in "Random structures and algorithms". The main result is proved in Theorem 1 and is an analogue of CLT, where the limit random variable is actually a maximum of two independent normal random variables. Work [6] is jointly with Ts. Zuevski and O. Kunchev and is published in "Chaos, Solitons and Fractals". Two different schemes in case of default are considered and two differential equations are obtained for the price of the respective default derivatives. The first is related to a stochastic differential equation by geometric Brownian motion, and the second has added jumps, with the stopping time being the moment of the first jump. The main results are proved in two theorems. Interesting examples are considered and interesting computer graphics are presented.

Everything stated in this section of the review gives me reason to conclude that the scientific output and scientific-metric indicators of the candidate are at a high level and undoubtedly fully meet all the conditions for holding the required academic position "professor".

III. Opinions, recommendations and notes

I have been following the scientific development of Mladen Savov for a long time and I have excellent impressions of his reports at seminars and international conferences. I would especially like to mention his participation in the National Seminar on Stochastics, of which I am also chairman. In addition, I was a reviewer of his dissertation (in English) for "Doctor of Science", defended at the Institute of Mathematics and Informatics of BAS. I would like to give part of the conclusion of the review:

«The presented dissertation has all the qualities of a serious monographic work in the field of Stochastics. The obtained interesting results give complete solutions to a number of issues related to the exponential functionalities of Levy. A number of new methods have been developed, both in analytical and stochastic aspects. Of particular interest are the Bernstein-Gamma functions and their stochastic applications.

All these results have been published in prestigious publications with a high impact factor in the last 7 years, but they already have a very serious international response (over 111 citations). In addition, they have been reported at a number of prestigious international conferences and symposia (some by invitation and as plenary reports), as well as at seminars at a number of renowned universities. »

And here I would like to make a remark to the candidate that he did not attach to the materials of the competition the abstract from this dissertation. The same goes for his previous (PhD) dissertation. This would give a greater completeness of the presentation.

On the other hand, I would like to emphasize his correctness, responsiveness and empathy for the problems of individual colleagues and the scientific community in general, as well as the respect he enjoys, both at home and abroad.

I would recommend keeping the high level in the scientific development and also trying to attract MD and PhD students from FMI in the interesting area of his research.

Conclusion

The inspection of the submitted materials for the promotion (competition) did not reveal any violations in the procedure, as all the above-mentioned requirements were met.

As it has already been emphasized in the previous sections, the scientific production of Mladen Savov is at a high scientific level, as well as his scientificmetric indicators. He is a "Doctor of Science" in the field of promotion and has appeared as a participant and leader of scientific and applied research projects, both in Bulgaria and internationally. His teaching activities, especially at Sofia University St. Kliment Ohridski, undoubtedly deserve admiration. M. Savov is an internationally recognized specialist in the field of Stochastics with publications in a number of prestigious journals with a high impact factor. In general, we can conclude that his scientific production covers a wide range of stochastic processes and is at a high scientific level.

Everything presented up to now gives me a definite reason to conclude that the only candidate, Associate Professor **Mladen Svetoslavov Savov**, Dr.Sci., undoubtedly satisfies all the conditions of the announced promotion for the academic position of "Professor" at the Faculty of Mathematics and informatics of Sofia University "St. Kl. Ohridski » and I call on the scientific jury and the faculty council to vote positively for his election.

Date: 06.08. 2020

REVIEWER:

/ Professor Nikolay M. Yanev/