Report about the thesis submitted by Anton Zinoviev to Sofia University

1 Title

Finite Models by Resolution with Termoids

2 Referee

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3 Subject

The thesis of Anton Zinoviev falls within the framework of the classical decision problem and the method of resolution. Approaches based on the method of resolution are multifarious in theoretical computer science and their study has given rise to many mathematical questions such as soundness, completeness and termination. In this setting, an important issue concerns the use of the method of resolution for deciding solvable fragments of first-order predicate logic and constructing models of satisfiable formulas in these fragments. See [Caferra and Zabel (1991)], [Formüller and Leitsch (1996)] and [Tammet (1992)] for informations on that subject. For deciding solvable fragments and constructing models of their satisfiable formulas, an important property can be helpful: the finite model property. A fragment of first-order predicate logic possesses the finite model property when its satisfiable formulas are satisfied in finite models. One of the fragments of first-order predicate logic that possess the finite model property is the VED fragment consisting of finite sets of Horn clauses in which individual variables always occur at the same depths. The VED fragment can be decided by the use of the approach of the positive hyperresolution for constructing finite models. See [Caferra *et al.* (2010)] for informations on that subject.

Anton Zinoviev aims to contribute to the issue concerning the use of the method of resolution for deciding solvable fragments and constructing models of their satisfiable formulas. In this setting, he has taken the following challenges: (i) definition of termoids as a generalization of terms; (ii) definition of unification for termoids; (iii) presentation of the method of resolution and the approach of the positive hyperresolution for termoids; (iv) presentation of gamma-termoids, delta-termoids and epsilon-termoids, again as a generalization of terms; (v) definition of unification for delta-termoids and epsilon-termoids; (vi) proof of the finite model property for the VED fragment. To achieve his aim, Anton Zinoviev defines, for each kind of termoids above-mentioned, the corresponding

notion of substitution, the corresponding unification procedure, the corresponding method of resolution and the proof of its completeness. These definitions give rise to multifarious theorems that necessitate the consideration of many intermediate lemmas and propositions as well as the examination of many different cases in their proofs.

4 Structure

The thesis of Anton Zinoviev is 245 pages long. It comes with an abstract of 31 pages. Since a new terminology is used to present the algebraic theory of termoids, this abstract hardly helps the reader to grasp the ideas and concepts that will be considered in the thesis. Nevertheless, it constitutes a coherent set of information that faithfully describes the principal results of the thesis. The thesis itself consists of 6 chapters preceded by an introduction and followed by a conclusion. The list of the 2 publications of Anton Zinoviev that are related to the thesis is given at the end of the conclusion. It comprises a publication in a journal (Annuaire de l'Université de Sofia).

5 Content

Chapter I presents in an informal way the method of resolution and its use for deciding solvable fragments of first-order predicate logic and constructing models of satisfiable formulas in these fragments. It also introduces termoids, a generalization of terms and, in his thesis, Anton Zinoviev introduces several kinds of termoids. Finally, Chapter I presents the method of resolution and the approach of positive hyperresolution for termoids. All in all, this chapter constitutes a nice introduction to the tools and techniques that will be developed in the following ones. It also presents, without proving them, some of the important results of the thesis that will be studied later.

Chapter II constitutes the real beginning of the thesis. It presents the syntax and the semantics of the first-order predicate logic. In his thesis, Anton Zinoviev has decided to follow a multi-sorted approach and the reasons to do so are given in the first section of Chapter II. But a more important and radical choice of presentation has been made in this chapter. It concerns the terminology used to develop the syntax and the semantics of the first-order predicate logic. In order to develop his theory of termoids, Anton Zinoviev has decided to present terms and formulas in a new way that is made for preparing the reader with the terminology used to develop termoids and formuloids in the forthcoming chapters. This choice of presentation may have its own merits, but it does not always help the reader in understanding ideas and concepts at stake, even the classical ones concerning the syntax and the semantics of the first-order predicate logic.

With Chapter III, Anton Zinoviev introduces the algebraic theory of termoids. Termoids are a slight generalization of terms, seeing that terms are equivalent to a special kind of termoids (the alpha-termoids), the main difference between terms and termoids being that, in each structure, a term has exactly one value whereas a termoid may have many values. In order to illustrate the connection between terms and termoids, Anton Zinoviev, in Section 16, shows how the set of terms can be embedded in the set of termoids. Finally, Anton Zinoviev considers termoid identities as identities between termoids and defines what are the substitutions that can solve them.

In Chapter IV, Anton Zinoviev introduces the method of resolution for termoids. Such method consists in trying to refute a finite set of clausoids. Clausoids are finite disjunctions of literaloids, that is to say finite disjunctions of literals based on termoids. Then, in Section 21, Anton Zinoviev introduces the well-known method of SLD-resolution, this time within the context of termoids. In Sections 22 and 23, the approach of the positive hyperresolution is introduced for clausoids and is proved to be sound and complete. Positive hyperresolution is not sound for arbitrary structures, but Anton Zinoviev proves its soundness in the case of structures of terms.

Chapter V is the longest (70 pages) chapter of the thesis. It contains the definitions of different species of termoids: the gamma-semitermoids, the gamma-termoids, the delta-semitermoids, the delta-termoids, the epsilon-regular delta-termoids and the epsilon-termoids. In this chapter, Anton Zinoviev explains what unification is within the contexts of delta-termoids and epsilon-termoids. Strangely, the last paragraph of this chapter is a definition.

Finally, in Chapter VI, the finite model property of the VED fragment of first-order predicate logic is proved. More precisely, in this chapter, Anton Zinoviev shows the following: (1) if a termal system of identities is unsolvable then it is unsolvable in an algebra with a finite carrier-set; (2) if a finite set of clausoids is satisfiable in almost any algebra then it is satisfiable in an algebra with a finite carrier-set. The property (1) is proved in Section 29 as Theorem C. A weaker version of it has been proved in the 1960s by Gladstone. Theorems L and M extends the property (1) to delta-termoidal systems of identities and epsilon-termoidal systems of identities. Concerning the VED fragment, Anton Zinoviev shows that it possesses the finite model property in Section 30 as Theorem W.

6 Evaluation

The thesis of Anton Zinoviev is original and contributes to the development of new tools and techniques in the use of the method of resolution for deciding solvable fragments of first-order predicate logic and constructing models of satisfiable formulas in these fragments. His approach is based on termoids and clausoids, a generalization of terms and clauses, and the associated definitions

of substitution, unification, resolution and hyperresolution for them. Anton Zinoviev skillfully uses these classical tools and techniques that have been developed during the years within the framework of the classical decision problem. He shows in this respect his ability in the field of automated reasoning in the first-order predicate logic. I have been able to check many arguments displayed in the proofs of the thesis' results. Some of them are really difficult and necessitate the consideration of many intermediate lemmas and propositions or are based on arguments done *by induction*.

But many of the thesis' results are equivalent to simple remarks whereas their proofs consist of long sequences of uninteresting verification steps. In some other cases, the reader of the arguments displayed in the proofs is hardly convinced of their correctness. For instance, the use of the König's Lemma in the proof of Proposition in Section 18. Other arguments that hardly convince the reader of their correctness are those used in the proof of Lemma I in Section 19 or in the proof of Lemma G im Section 21. Lemma G is surely a well-known property and its proof has probably been already given and published in the past. There are also problems in the definitions of the new tools proposed by Anton Zinoviev. I understand that defining new tools necessitates the extension of old terminologies. But there is no need to completely change the well-known existing terminologies while developing the theory of termoids. The result of the choice made by Anton Zinoviev at Page 38 is that even Chapter II, which is about the well-known syntax and semantics of first-order predicate logic, can be hardly read by a professional logicians.

7 Conclusion

Putting apart the above-mentioned defects in the presentation of the thesis, the whole picture of the document presented by Anton Zinoviev contributes to the development of new tools and techniques in the use of the method of resolution for deciding solvable fragments of first-order predicate logic and constructing models of satisfiable formulas in these fragments. Therefore, it constitutes a rather coherent set of results in Mathematics (Mathematical Logic). For this reason, I consider that Anton Zinoviev deserves to receive the title of Doctor in Mathematics (Mathematical Logic) of Sofia University.