

# Efficient calculations and algorithms. Applications in solving theoretical and practical problems from various scientific areas.

Research Group: Modern Technologies  
Research field: Informatics

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## Learning Families of Algebraic Structures from Text

**Research group:** Nikolay Bazhenov, Ekaterina Fokina, Dino Rossegger, Alexandra Soskova and Stefan Vatev

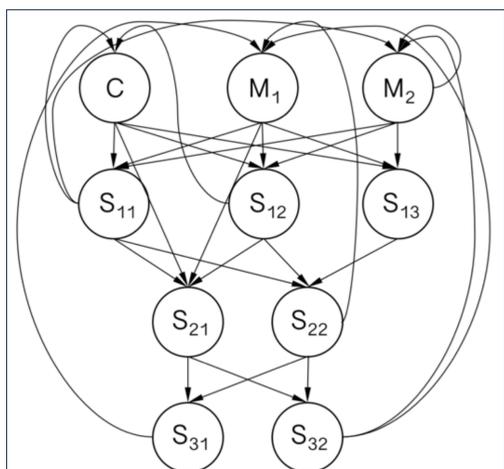
We adapt the classical notion of learning from text to computable structure theory. Our main result is a model-theoretic characterization of the learnability from text for classes of structures. We show that a family of structures is learnable from text if and only if the structures can be distinguished in terms of their theories restricted to positive infinitary  $\Sigma_{\aleph_2}$  sentences.

## Efficient generation of synthetic data representing RNA sequencing

**Research group:** Petar Armyanov, Ivan Ivanov, Dafina Petkova, and Nikolay Shegunov

Analyzing data retrieved from real cell RNA sequencing is quite expensive. Such data consists of values for the expression of thousands of genes, making it difficult to draw conclusions about any relation among the genes and to build valid gene regulatory networks. In our study, we start with formulating a hypothesis for possible gene relations and regulatory hierarchy and then generate synthetic data that would be expected if the hypothesis is true. This data is then compared to observations from an experiment to either accept or reject the hypothesis with a certain degree of confidence. As the data size grows exponentially with the number of studied genes, tasks such as data generation, analysis and result comparison require enormous computing power. Specially designed algorithms have to be implemented and then executed on high-performance computing clusters, which is the main focus of this study.

The picture below shows a synthetic boolean network representing relations among ten genes.



## Non-commutative theory of invariants.

**Research group:** Silvia Boumova, Deyan Dzhundrekov

Noncommutative invariant theory began with the work of Margaret Wolf in the first half of the 19th century on noncommutative symmetric polynomials. Subsequently, the theory developed a lot, generalizing many of the classical results, many of which look different in the non-commutative case. Recent years have seen a return to the study of concrete algebras of invariants, calculation of Hilbert series, finding generating elements, determining ratios, and more. The goal of our research is to find the generating elements of the algebra of the invariants of the free algebra of  $d$  variables, under the action of some group (dihedral, alternative, and others) and under the additional Koryugin action.

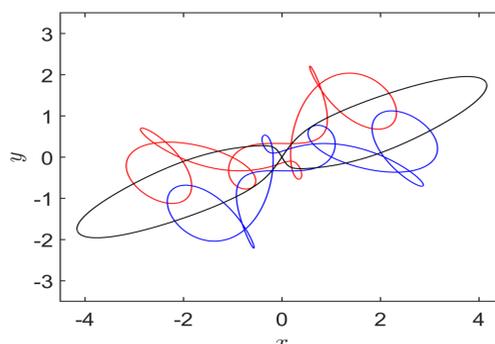
The case of the symmetric group is solved and the results are published in two papers. Cases depending on the characteristic of the main field should be considered. Our goal is to replace the symmetric group with the alternative group.

## Improvement of numerical algorithms for searching for new periodic planar three-body orbits

**Research group:** Ivan Hristov, Radoslava Hristova, Petar Armyanov, Nikolay Shegunov

We are interested in the periodic orbits of the classic gravitational three-body problem. The stable periodic orbits are astronomically very important and are not so difficult to be found numerically, but all periodic orbits are important for the theory. Particularly, the unstable periodic orbits give us a good knowledge of the system and provide crucial information in chaotic regions. Hence, one of the important tools for the study of chaos for this problem is periodic orbit search algorithms. Although thousands of new periodic orbits have been discovered in the last decade, search algorithms are far from exhaustive and need to be improved. This is one of the goals of our research.

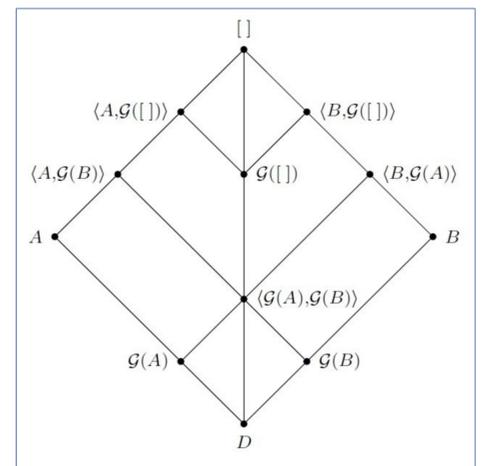
The picture below presents the plot of one of the many found new unstable periodic solutions.



## Subrecursive representations of irrational numbers

**Research group:** Ivan Georgiev, Lars Kristiansen

We investigate the structure of the subrecursive degrees of representations of irrational numbers with respect to subrecursive reducibility. We say that a representation  $R_1$  is subrecursive in a representation  $R_2$ , if given any  $R_2$ -representation of an irrational number  $\alpha$ , we can produce an  $R_1$ -representation of  $\alpha$  by an algorithm with no unbounded search. For example, let  $\mathbf{D}$  be the Dedekind cut, which given a rational  $q$  decides whether  $q < \alpha$  or  $q > \alpha$  and let  $[ ]$  be the continued fraction, which given  $n$  returns the  $n$ -th partial quotient  $a_n$  of  $\alpha$ . Then the representation  $\mathbf{D}$  is subrecursive in  $[ ]$ , but  $[ ]$  is not subrecursive in  $\mathbf{D}$ . In fact, a recent paper by Georgiev shows a profusion of subrecursive degrees, which lie strictly subrecursively between  $\mathbf{D}$  and  $[ ]$ :



One of the main goals of the present project is to enrich the knowledge about different complexity classes of irrational numbers: 1) by exploring the subrecursive degrees of new representations and 2) by considering the graphs and the boolean combinations of already known representations.

## Conclusion

The main goal of the project is to share knowledge from different, but closely related science areas and build collaboration among experts in mathematics, physics, biology and computer science. This will be achieved with joint research in the fields of computability theory, algebra and numerical methods in order to address different problems in natural science, such as studying the unstable orbits of material bodies, gene regulatory networks and fundamental mathematical properties of floating point calculations.