R E P O R T

by **Prof. Stefan Ivanov**

on a Thesis for awarding the degree "Doctor of Sciences"

Title: **Finite Geometries and Codes**

Author: **Assoc. Prof. Assia Petrova Rousseva**

Scientific field: 4. Natural sciences, mathematics and informatics

Professional field: 4.5. Mathematics

Overview of the thesis

The presented thesis amounts 180 pages of text. It consists of five chapters and a list of references including 201 items. The thesis is summarizes and generalizes the research by the author on several problems in the field of finite geometry closely related to optimization problems from the theory of the linear error-correcting codes. It contains results on the achievement of the classical Griesmer bound, results on the extendabilty of linear codes and of arcs in finite projective geometries, as well as new constructions of affine blocking sets meeting the lower bounds by A. Bruen and S. Ball.

Description of the main results

The original results are contained in chapters 3, 4, and 5.

Chapter 3 is focused on the achievement of the Griesmer bound. According to a result by Dodunekov this bound gets worse than the sphere-packing bound for high dimensions. The author investigates the deviation of the length of the optimal codes from this bound. In Theorem 3.10 she generalizes a result by Belov, Logachev, and Sandimirov, which allows to assess the rate of this deviation.

The same problem for three-dimensional codes was proposed by S. Ball. He formulates a hypothesis which is proved in the thesis (Theorem 3.18) for desarguesian planes of even order. At the end of the chapter, the author proves the non-existence of several hypothetical Griesmer codes over the field with four elements by using geometric techniques.

In **Chapter 4** the candidate considers the extendability problem for codes, which is formulated as a problem for arcs in the geometries $\mathrm{PG}(r, q)$. She proposes a general approach in which the extendability problem reduces to a structure problem for a special class of arcs, the so-called $(t \mod q)$-arcs. Several characterization results on $(t \mod q)$-arcs are proved,

the most important of which is Theorem 4.12. It says that in geometries of prime order the vector space of all $(0 \mod q)$-arcs is generated by the complements of the hyperplanes. A central result in this chapter is also Theorem 4.27, which gives a sufficient condition on the extendability of Griesmer arcs with t-quasidivisibility. Using the developed techniques, the author rules out the existence of one of the four hypothetical optimal codes in dimension 4 over the field with five elements.

The last **Chapter 5** contains new constructions of good blocking sets in finite affine geometries meeting the lower bounds of Bruen and Ball. The main result here is Theorem 5.6 which is a general construction for affine blocking sets. As a special case, the author obtains a new infinite class of blocking sets meeting Bruen's bound. This is only the third infinite class of blocking sets meeting this bound, and in fact the second non-trivial class. The general construction gives five examples of blocking sets meeting a new lower bound proved by Ball in 2014. These are the first and only examples of blocking sets meeting this bound.

Publications related to the thesis

The results of this thesis are published in seven scientific papers five of which have not been used in previous procedures. The remaining five papers are published as follows:

- *Designs, Codes and Cryptography*, IF:1.224 (Q2,2018),

- *Advances in Mathematics of Communications*, IF:0.8 (Q3,2016),

- *Comptes Rendus de l'Academie bulgare des Sciences*, IF:0.321 (Q4,2018).

Two of the remaining papers are published in *Annuaire de l'Universite de Sofia*. In two of the publications Assia Rousseva is the only author. In the remaining three she has one coauthor. I accept that the contribution of Dr. Assia Rousseva is significant in all the presented publications.

The candidate presents a list of 8 citations of her results in papers appearing in refereed journals.

The results of this dissertation thesis were presented at numerous scientific conferences and workshops worldwide: ALCOMA (Germany, 2010, 2015), WCC (France, 2015, 2019), Combinatorics (Italy, 2012, 2014, 2016, 2018), Finite Geometries (Kloster Irsee, 2014, 2017), Finite Fields and Their Applications (Italy, 2017) and many others.

Remarks and comments

One of the central results in chapter 4, Theorem 4.12 is proved only for prime fields. Is it possible to extend it to arbitrary finite non-prime fields?

Author's summary

The author's summary is made according to the regulations and reflects properly the main results and contributions of this thesis.

Conclusion

The proposed thesis, the summary, the scientific papers and the related other documents show that Assia Rousseva is an established mathematician in the field of finite geometry and coding theory with significant contributions in these two areas of research. The dissertation thesis contains results that are important theoretically and uncover new aspects of the relation between coding theory and finite geometry. The candidate demonstrates deep knowledge of his field and the capacity to develop it in a new and important way. With this, she meets the national requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its applications and the specific Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in Sofia University. The scientific papers do not repeat these of previous procedures. There is no evidence of plagiarism in the presented scientific texts.

Based on the above analysis, I assess **positively** the presented Thesis "Finite Geometries and Codes" and recommend that this panel awards **Assia Petrova Rousseva** the scientific degree "Doctor of Sciences" in the scientific field 4. "Natural sciences, mathematics and informatics", professional field 4.5 "Mathematics".

Sofia, 20.06.2020 Member of the Scientific Jury:

(Prof. Stefan Ivanov)