Review

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about the thesis

"On some Diophamtone equations and inequalities"

by Zhivko Hristov Petrov

for awarding the educational and scientific degree "Doctor of Philosophy"

Area of higher education: 4 "Natural Sciences, Mathematics and Informatics",

Professional field: 4.5 "Mathematics",

Doctoral program "Mathematical analysis"

Scientific advisor: Prof. Doychin Ivanov Tolev, Dr. Sc.

The thesis is 93 pages long and consists of five chapters and a bibliography. There are 88 citations, all of them from a specialized literature. The first chapter is introductory, the basic consepts are introduced there and the basic results are formulated. In the second chapter are given some known theorems and lemmas from mathematical analisys and number theory, which are used in the proofs of the theorems of the thesis.

The third chapter is devoted to the equation

$$[p^c] + [m^c] = N, \tag{1}$$

where $c \in (1, 29/28)$ is a constant and $[t]$ denotes the integer part of the real number t. It is proved that if N is a sufficiently large natural number then (1) has a solution in unknows p and m, where p is a prime and m is a natural number with no more than $\left[\frac{52}{29-28c}\right] + 1$ prime factors. This theorem is similar to the famous theorem of Chen, which states that every sufficiently large even number can be represented as a sum of a prime and an integer with no more than two prime factors. However here the unknowns are risen to a power greater than 1 (though near to 1), wich makes the problem more difficult. That is why the restriction for the number of prime factors of m is less severe than in Chen's theorem. It would be interesting to find a version of the present theorem in which m has no more than two prime factors.

The result of this chapter was obtained jointly by the autors of the thesis and his scientific advisor (and author of the present review). It was published first as a preprint in ArXiv, which was cited in "Acta Arithmetica", and then it was published in the "Proceedings of the Steklov Institute of Mathematics".

The fourth chapter is devoted to the equation

$$[p_1^c] + [p_2^c] + [p_3^c] = N, \tag{2}$$

where $c \in (1, 17/16)$ is a constant. It is proved that if N is a sufficiently large natural number then the equation is solvable in prime numbers p_1, p_2, p_3, such that each of the numbers p_1+2, p_2+2, p_3+2 has no more than $\left[\frac{95}{17-16c}\right]$ prime factors. The equation (2) is an analog of the ternary Goldbach equation and it was explored first by Laporta and the author of the reiew, but without restrictions on p_i, and then also by other mathematicians. I'll note that the restriction on the number of prime factors of p_i+2 introduces additional complications. The result of this chapter was published by the author of the thesis in "Annual of Sofia University".

The subject of the last fifth chapter is the Diophantine inequality of Piatetski-Shapiro

$$|p_1^c + \cdots + p_s^c - N| < (\log N)^{-1}, \tag{3}$$

where $c > 1$ is a fixed number, which is not an integer, $p_1, \ldots, p_s$ are prime numbers belonging to the Piatetski-Shapiro set $\mathcal{N}_\gamma$ and $\gamma \in (0, 1)$ is a parameter. There are four theorems related to the solvability of the inequality in primes of that type, provided that N is a sufficiently large real number and the parameters c and γ satisfy certain conditions. The first theorem is related to the case when c is large and γ is near to 1. It states that if s is sufficiently large with respect to c and if N is a sufficiently large real number, then (3) has a solution in primes $p_1, \ldots, p_s \in \mathcal{N}_\gamma$. In the next two theorems the cases $s = 4$, $s = 3$ are considered. It is proved that if and c and γ are close to 1 then (3) has a solution in primes of the type mentioned above for all sufficiently large N. In the last theorem the case $s = 2$ is considered, and again c and γ are close to 1. It is proved the solvability of the inequality for almost all large N (in the sence that the measure of the set of the numbers N for which the inequality has no solution in primes $p_1, p_2 \in \mathcal{N}_\gamma$, is small).

These results are obtained jointly by the author of the thesis and by Prof. Angel Kumchev (Towson University, USA) and are published in „Monatshefte für Mathematik".

The subject of the dissertation is up to date. The problems solved are classical and are similar to problems solved by prominent mathematicaians. The proofs in the thesis are detailed and clear, the historical reference and the bibliography are complete. The autor's abstract of the thesis is written according to the requirements of the rules and completely describes the contributions of the thesis. As I already mentioned, the results of the thesis are published in three articles, two of them in foreign journal with high IF ("Proceedings of the Steklov Institute of Mathematics", „Monatshefte für Mathematik") and the third is published in "Annual of Sofia University". One of the papers is already cited in "Acta Aritmetika".

Zhivko Petrov has reported his results on the international conference "Journees Arithmetiques" in 2017 in France; on three spring scientific session of the Faculty of Mathematics and Informatics in 2016, 2017 и 2018 ; on the session of the section "Algebra and Logic" of the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences in 2016 as well as in several lectures of the seminar "Dynamical systems and number theory" in the Faculty of Mathematics and Informatics.

I'll also note that, as a scientific advisor of Zhivko Petrov, I have watched his work and my impression is that he has entered the subject of Analytic Number Theory and that he is ready independently to carry out scientific investigation on this field in future. His knowledge of mathematical analysis is at the required level as well. I also have excellent impressions on pedagogic skills of the autor of the thesis. During the last years we have worked together — I have read lectures, and he has held seminars.

Conclusion:

In my opinion the present thesis completely meets the requirements of the law on the development of the academic staff in the Republic of Bulgaria, as well as the specific requirements of the Faculty of Mathematics and Informatics of Sofia University. This gives me the reason to propose the educational and scientific degree "Doctor of Philosophy" to be awarded to Zhivko Hristov Petrov in the area of higher education 4 "Natural Sciences, Mathematics and Informatics", professional field 4.5 "Mathematics", doctoral program "Mathematical analysis".

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Prof. D. Tolev