

Review

of the thesis for obtaining the educational and scientific degree "Doctor" in the professional field
4.5 "Mathematics" (Mathematical modeling and application of mathematics in the economy)

Author of the dissertation: Nikolay Kostov Chervenov

Thesis theme:
Copulas in Sobolev spaces and applications .

Author of the review:
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The copula theory is relatively young. The notion and the term copula appeared in 1959. Later, the interest in this theory grew explosively. The explanation is in the following quote from the book by Durante and Sempi, which is used in the dissertation: "... As we have seen, the notion of copula is both natural as well as easy for looking at multivariate distribution functions. But why do we witness such incredible growth in papers published starting the end of the nineties. Here I can give three reasons: finance, finance, finance. "

The actuality and the significance, both theoretical and practical, of the dissertation theme is beyond doubt.

What are the copulas? - A probabilistic model has, so to say, two components - one is the set of random variables in the model and the other is the dependencies between them, the orderly pattern of the model. The copula is a function that represents this second component – it is a function that creates a probabilistic model from the individual random variables, linking them. Formally speaking, the copula is a multivariate function, such that if we substitute its arguments for the distribution functions of the random variables of the model, we get the joint distribution function of these variables.

A basic idea in the dissertation is to use the tools of the differential equations theory and the functional analysis for the study of the copulas. The problems related to the copulas and their properties are formulated in a natural way in terms of the partial differential equations theory. Generally speaking, this allows for some simplifications, generalisations and, above all, general methods, avoiding the need for specific constructions for each particular case. The class of functions that can be copulas is enlarged – the copulas are considered as distributions in the sense of Schwarz, elements of Sobolev spaces. A generalisation of the characteristic for copulas property of "n-increasing" is made, which facilitates the verification of this property. This is illustrated by the example of numerous well-known copulas, and the important class of archimedean copulas is thoroughly examined.

The problem of copulas building is reduced to a boundary value problem for a partial differential equation that the author calls Goursat problem. This is the "copula equation" and the thorough study of this problem is the main contribution in the dissertation. A priori estimates, theorems of existence and uniqueness of solutions, as well as their qualitative characteristics (smoothness, singularities) were obtained. Important are the deduced restrictions on the right-hand side of the equation that imply the boundary conditions of the problem. It is worth mentioning the good knowledge and skillful use of the tools of the functional analysis. The following question arises: when the copula, being a solution to this boundary value problem, is a distribution in the sense of Schwartz which has no values, how should we determine the joint distribution function (of the probability model)?

The last chapter of the dissertation is a practical application of the proposed method for building copulas. Using this method the relationship between the amount of claims and the moment when they occur for a Bulgarian insurance company has been examined.

In summary, the main contributions in the dissertation are:

- A natural generalization of the problem of building copulas has been found, which has been extensively studied. Relevant results and methods, that also have practical application, have been obtained.
- Relevant generalizations of the "n-increasing" property have been found and have been applied to an important class of copulas.
- A case of the Bulgarian insurance practice has been investigated by applying the above method.

There are 6 publications related to the dissertation - all joint with other authors.

There is no citation and impact factor data.

The results of the dissertation are presented in three conferences.

The synopsis is well written and correctly reflects the content of the dissertation.

I do not know the author and I have no personal impressions of him.

I have no substantial critical remarks. There are some typographical errors - I point one: on the last line on page 84 and the first line on page 85 there is a combination of words that is incomprehensible.

I would recommend to the author to examine in the light of his findings, the use of copulas for the study of Markov processes done by Darsow et al.

Conclusion

I consider that the dissertation meets the requirements of the Law (ЗПАЧРБ) and I recommend to the honorable jury to award Nikolay Kostov Chervenov the educational and scientific degree "Doctor" in the professional field 4.5 "Mathematics" (Mathematical Modeling and Application of Mathematics in Economics).

April 8, 1919

Signature: prof. Racho Denchev