Review of Doctoral Thesis

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Title: **Copulas on Sobolev spaces and Applications**
Reviewer: **Prof. Leda Minkova, PhD, DSi**

Nikolay Chervenov graduated in bachelor program in mathematics and master program in mathematics in Sofia University. By student exchange program Erasmus, he was one year in the University of Nantes, France. During the last years he was a doctoral student in the department of Probability, Operations Research and Statistics in Sofia University. He has a research interest in Actuarial Sciences.

General description. The thesis consists of Introduction, three chapters, Conclusion and a bibliography, written on 98 pages. The bibliography contains 69 titles. The thesis is presented for acquiring the educational and scientific degree "Doctor"in the PhD study program Mathematical Modeling and Application of Mathematics in Economics.

In the Introduction it is given a review of the literature related to copulas. During the last two decades the authors work intensively on the copulas and their applications. It is well known that the independence is not a real assumption in practice. In this reason, all models with dependency are useful. The copula presents a dependence between marginal probability distribution functions. Here, a family of copulas is constructed on the basis of stochastic methods and methods of partial differential equations.

In Chapter 1 the bivariate copulas are analyzed. Two definitions of 2-increasing function are given with their equivalence (Lemma 1.1.4). It is given a method for construction a family of copulas as a solution of a boundary value problem in a Sobolev space. The method reduces the checking the 2-increasing of C-volume of copula to simple differentiation. The application is demonstrated on a number of examples.

The main result in this chapter is a Theorem 1.2.6. An upper bound of the copula function is given. The continuous related to the right hand side function in the problem (1.2.20) is proved. In the subsection 1.3 the existence of the solution is shown. The general solution with the assumption of existence of weak derivatives is given in Theorem 1.4.1.

The Theorem of Sclar is a particular case of these results. A family of copulas, generated of function h, not given in the book of R.Nelsen, is constructed.

The results of this chapter are published in [35], the short version and [36], the extended version.

In Chapter 2 the notion of n-increasing property is generalized with the assumption of existence of weak derivatives. In this extension case, again two definitions of n-increasing function are given with their equivalence (Lemma 2.1.8). The prof in the case of n-dimensional copulas is on the basis of s.c. mean function, see (2.1.8). A special attention is paid to the Archimedean copulas (subsection 2.2). Some properties of the mean function are given. A method of obtaining a family of copulas as a solution of boundary values problem in Sobolev space is presented. The main result in this chapter is the Theorem 2.3.1. As application it is proved that the n-dimensional Archimedean copulas are n-increasing functions.

The author found that the Goursat problem is closely related to the construction of copulas. The result of Vladimirov is extended to the n-dimensional case. A theorem of existence and uniqueness of the solution to the Goursat problem is proved.

The results are published in [5], [67] and [68].

In **Chapter 3** a numerical example is given. The data are 5 years CASCO motor insurance claims. The dependence between the claims and the time to the claims is analyzed. A review of the numerical methods for such type of problems is given. In this case, the author applies a spectral numerical method on the basis of Chebyshev polynomials. The resulting numerical copula fits the data well. Actually, the numerical copula describes well the real situation. The resulting copula is very close to the independent copula. This is shown by the four dependence coefficients, which are equal to zero. My advise to the author is to continue working on the application of the method. The result would be interesting if the data are with strong tail dependence.

The results are published in [69].

In the **Summary** the results obtained in the thesis are given. I think the results of the thesis mentioned of the author are true.

Publications The thesis is written on the basis of 5 published papers and one submitted paper. The submitted paper is not presented in the documents. Two of the papers are in journal with IF. The results of the thesis are reported during one conference and two spring scientific session in the Faculty of Mathematics and Informatics.

Conclusion. The thesis represents a great deal of work. The results are

well presented and their interpretation and application are at a high scientific level. In my opinion, the thesis by Nikolay Chervenov fulfils the conditions for gaining the PhD degree in Mathematical Modeling in Economics. Therefor, I strongly recommend the thesis of receiving the Degree of PhD.

Sofia, April 11, 2019 Leda Minkova