Report about the thesis submitted by Vladislav Nenchev to Sofia University

Title

Region-Based Theories for Space and Time: Dynamic Relational Mereotopology.

Referee

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Subject

The thesis of Vladislav Nenchev falls within the framework of the research dedicated to developing Whitehead's ideas about point-free theories of space and time. Whitehead's ideas have been recently rediscovered and developed in areas like knowledge representation and qualitative reasoning. It is a topical subject that has many possible applications in computer science and computational linguistics (geographical information systems, natural language understanding, etc) and where there are many mathematical questions still to be answered (expressive power, axiomatization, completeness, decidability, etc). In this setting, following the line of reasoning suggested by [Randell *et al.* (1992)] and furthered by [Wolter and Zakharyaschev (2000)], one usually considers, in a given topological space, the Boolean algebra of its regular closed subsets together with mereotopological relations such as external-contact, overlap, tangential-part-of, non-tangential-part-of, etc. Given a first-order language interpreted over these mereotopological relations, the main question is the decidability and complexity of the associated satisfiability problem. See [Kontchakov *et al.* (2013)] for a recent paper on that subject. Another line of research that has a bearing on Whitehead's ideas attempts to establish correspondences between point-free mereotopological models such as Boolean algebras with contact relations and point-based topological spaces. The main task, here, is to determine necessary and sufficient properties of the relations that constitute these mereotopological models in such a way that we can equivalently represent them as binary relations defined in topological spaces. See [Düntsch and Winter (2005)] and [Dimov and Vakarelov (2006)] for important papers on that subject.

Vladislav Nenchev aims to contribute to the definition of relational systems consisting of spacetime regions together with mereotopological relations such as *part-of*, *overlap*, *underlap* and *contact*. In this setting, he has taken up the following challenges: (i) representation theory of the static relational mereotopology and mereology; (ii) representation theory of the dynamic relational mereotopology and mereology; (iii) decidability and complexity of first-order theories interpreted over these static and dynamic mereotopologies and mereologies; (iv) axiomatization, completeness, decidability and complexity of modal logics interpreted over these static and dynamic mereotopologies and mereologies. To achieve his aim, Vladislav Nenchev proposes an approach based on classical tools and techniques such as representation theory by means of prime filters and prime ideals, Henkin-style construction, p-morphism lemma, relative interpretability, filtration technique, etc. The use of these tools and techniques allows him to prove difficult results such as representation theorems of the static and dynamic mereotopologies and mereologies, complete axiomatization of the quantifier-free fragments of the first-order theories of the dynamic mereotopologies and mereologies, complete axiomatization of the modal logics of dynamic mereotopologies and mereologies, hereditary undecidability of some first-order theories of the dynamic mereotopologies and mereologies, NP-completeness of some quantifier-free fragments of the first-order theories of the dynamic mereotopologies and mereologies, finite model property and decidability of some modal logics of dynamic mereotopologies and mereologies.

Structure

The thesis of Vladislav Nenchev is 125 pages long. It comes with an abstract of 30 pages that methodically and truthfully describes the principal results of the thesis. The thesis itself consists of 7 chapters, the first one being an introduction about region-based space and time (Chapter I) and the last one being a conclusion (Chapter VII). The list of the publications of Vladislav Nenchev that are related to the thesis is given at the end of Chapter VII. It comprises 2 publications in journals (*Central European Journal of Mathematics* and *Logic and Logical Philosophy*).

Content

Chapter I presents the scientific context of the thesis: region-based theories of space and time. Firstly, it quickly describes some of the most important mereotopological and mereological systems that have been considered after Whitehead: Tarski's mereology, Leśniewski ontology, contact algebras, Region Connection Calculus, etc. Secondly, it quickly introduces the method that have been used in [Vakarelov (2008, 2010, 2012)] to define dynamic contact algebras as Cartesian products of static contact algebras.

Instead of giving a comprehensive description of theories for space and time, Chapter II only concentrates on the theories that are connected with the subject of the thesis (contact algebras, spatial logic $S4_u$, Region Connection Calculus, Linear Temporal Logic). Its intention is to present Vakarelov's method for combining space and time and to collate it with other tools and techniques for producing formalisms for dynamic systems.

Chapter III is the first chapter of the thesis to contain mathematical results. Firstly, following the line of reasoning suggested by [Nenov and Vakarelov (2008)], Vladislav Nenchev defines in every contact algebra its associated mereotopological relations of *part-of*, *overlap*, *underlap* and *contact*. Secondly, he defines static mereotopological structures by a finite set of elementary conditions these mereotopological relations satisfy. Thirdly, in order to prove that we can equivalently represent the mereotopological relations that satisfy these elementary conditions as binary relations defined in topological spaces, he defines, for each static mereotopological structure, filters, prime filters, ideals and prime ideals that will be used to construct the abstract points associated to it. Fourthly, by means of a Separation Lemma and an Extension Lemma that are used to expand filters and ideals to their prime counterparts, he proves a Representation Theorem for static mereotopological structures.

With Chapter V and Chapter VI, Chapter IV contains the main results of the thesis. In Chapter IV, Vladislav Nenchev takes up again Vakarelov's method for combining space and time. Firstly, he defines standard dynamic mereotopological structures as Cartesian products of static mereotopological structures. Secondly, he defines in every standard dynamic mereotopological structures its associated stable and unstable mereotopological relations of *part-of*, *overlap*, *underlap* and *contact*. Thirdly, he defines dynamic mereotopological structures by a finite set of elementary conditions these stable and unstable mereotopological relations satisfy. Fourthly, in order to prove that we can equivalently represent the stable and unstable mereotopological relations that satisfy these elementary conditions as binary relations defined in Cartesian products of static mereotopological structures, he defines, for each dynamic mereotopological structure, filters, prime filters, ideals and prime ideals that will be used to construct the abstract points and the abstract time moments associated to it. Fifthly, by means of the above-mentioned Separation Lemma and Extension Lemma, he proves a Representation Theorem for dynamic mereotopological structures.

In Chapter V, Vladislav Nenchev introduces first-order and modal languages to be interpreted in dynamic mereotopologies and mereologies. Concerning the first-order languages, they are based on the binary predicates $\leq$ and $\preceq$ for stable and unstable part-of, o and O for stable and unstable overlap, u and U for stable and unstable underlap and c and C for stable and unstable contact. Thanks to the above-mentioned Representation Theorems for dynamic mereotopological structures, axiomatic systems based on the first-order conditions $(M1)$–$(M30)$ and $(C1)$–$(C10)$ of dynamic mereotopologies and mereologies naturally give rise to complete first-order theories. Concerning the modal languages, they are based on the modal operators $[\leq]$, $[\geq]$, $[\preceq]$, $[\succeq]$, $[o]$, $[O]$, $[u]$, $[U]$, $[c]$ and $[C]$ — interpreted in dynamic mereotopologies and mereologies by the relation $\leq$, $\leq^{-1}$, $\preceq$, $\preceq^{-1}$, o, O, u, U, c and C — and the modal operator $[A]$ — interpreted in dynamic mereotopologies and mereologies by the universal relation. Apart from the condition $(M3)$ of antisymmetry for the stable relation of part-of, all of the first-order conditions $(M1)$–$(M30)$ and $(C1)$–$(C10)$ of dynamic mereotopologies and mereologies are modally definable with Sahlqvist formulas. In order to completely axiomatize the modal logics of dynamic mereotopologies and mereologies, Vladislav Nenchev introduces 3 first-order conditions $(M3')$, $(M3'')$ and $(M3''')$ for the stable relation of part-of and the unstable relations of overlap and underlap. These new first-order conditions hold in all dynamic mereotopologies and mereologies. Moreover, they are modally definable with Sahlqvist formulas. As a result, proving that every relational structures satisfying the conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M30)$ and $(C1)$–$(C10)$ is a p-morphic image of a dynamic mereotopology and proving that every relational structures satisfying the conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$ and $(M4)$–$(M30)$ is a p-morphic image of a dynamic mereology, Vladislav Nenchev obtains the completeness of his modal logics of dynamic mereotopologies and mereologies.

If Chapter V was about axiomatization/completeness of various first-order theories and modal logics for dynamic mereotopologies and mereologies, Chapter VI is about their decidability/complexity. Firstly, Vladislav Nenchev considers the full first-order theories of dynamic mereotopologies and mereologies and shows its hereditary undecidability by using the relative elementary interpretability of the first-order theory of a symmetrical irreflexive relation over a finite domain. Secondly, restricting the discussion to the quantifier-free fragments of the first-order theories of the dynamic mereotopologies and mereologies, he shows their decidability by proving a polysize model property of dynamic mereotopologies and mereologies: if a quantifier-free formula is satisfied in a dynamic mereotopology or mereologies then it is satisfied in a dynamic mereotopology or mereology of size linear in the size of the formula. As a consequence of this polysize model property, Vladislav Nenchev obtains the NP-completeness of the set of all quantifier-free formulas satisfiable in a dynamic mereotopology or mereology. Thirdly, via a filtration argument, he proves the decidability of the $[c]$-free modal logics determined by the first-order conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M30)$ and $(C1)$–$(C4)$ and the $\{[\preceq], [\succeq]\}$-free modal logics determined by the first-order conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M13)$, $(M17)$–$(M19)$, $(M23)$–$(M25)$, $(M29)$, $(M30)$, $(C1)$–$(C8)$ and $(C10)$. See below for more comments about these results.

Chapter VII consists of a a conclusion summing up the main results of the thesis, offering new ideas for its continuation and listing the publications of Vladislav Nenchev that are related to it.

Evaluation

The thesis of Vladislav Nenchev is original and greatly contributes to the development of Whitehead's ideas about point-free theories of space and time. His important results — representation theory of the static relational mereotopology and mereology, representation theory of the dynamic relational mereotopology and mereology, decidability and complexity of first-order theories interpreted over these static and dynamic mereotopologies and mereologies, axiomatization, completeness, decidability and complexity of modal logics interpreted over these static and dynamic mereotopologies and mereologies — has been obtained by an approach based on classical tools and techniques such as representation theory by means of prime filters and prime ideals, Henkin-style construction, p-morphism lemma, relative interpretability, filtration technique, etc. Vladislav Nenchev skilfully uses these classical tools and techniques and shows his strong capacity in rigorously proving difficult mathematical results. For example, the proof, in Chapter IV, of the Representation Theorem for dynamic mereotopological structures is 18 pages long and necessitates no less than 12 intermediate lemmas and propositions. Another example, in Chapter V, the use of the p-morphism lemma in the proof of the completeness of the modal logics of dynamic mereotopologies and mereologies necessitates the long and tedious verification of multifarious cases. In order to prove so many intermediate lemmas and propositions or to verify so many cases, one has, first, to have a thorough knowledge of the different tools and techniques used in this thesis and, second, to possess the mathematical skills that are required to rigorously apply them. And Vladislav Nenchev, certainly has this thorough knowledge and certainly possesses these mathematical skills.

I have been able to read and check the validity of the arguments displayed in the proofs of the thesis' results. They are almost always correct and typo-free (remind that some of them necessitate the proof of many intermediate lemmas and propositions or the verification of many cases) and Vladislav Nenchev succeeds in convincing the reader of their validity, especially when he is proving the Representation Theorem for dynamic mereotopological structures and the completeness of his modal logics of dynamic mereotopologies and mereologies. Nevertheless, he is less convincing when he is comparing, in Section 1 of Chapter V, the expressing power of different languages based on stable and unstable mereotopological and mereological relations or when he is discussing, in Section 1 of Chapter VI, about the hereditary undecidability of some first-order theories of the dynamic mereotopologies and mereologies. I believe that an appropriate rewriting of these sections will allow the reader to better evaluate the contributions of Vladislav Nenchev they encompass. Moreover, there are more serious problems in the Section 3 of Chapter VI that is devoted to the decidability, via filtration arguments, of the $[c]$-free fragment and the $\{[\preceq], [\succeq]\}$-free fragment of the modal logics of dynamic mereotopologies. The proof of Proposition 21 only shows that if a $[c]$-free formula is satisfiable in a relational structure satisfying the first-order conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M30)$ and $(C1)$–$(C4)$ then it is also satisfiable in a finite relational structure satisfying the same first-order conditions. Since there might exists finite relational structures that satisfy the first-order conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M30)$ and $(C1)$–$(C4)$ but that cannot be extended into relational structures that satisfy the first-order conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M30)$ and $(C1)$–$(C10)$, one cannot infer, from the above-mentioned finite model property, the decidability of the $[c]$-free fragment of the modal logics of dynamic mereotopologies, but the decidability of the modal logics determined by the first-order conditions $(M1)$, $(M2)$, $(M3')$, $(M3'')$, $(M3''')$, $(M4)$–$(M30)$ and $(C1)$–$(C4)$. This does not imply that the given proof of Proposition 21 is unreadable or erroneous (it is readable and 100%-correct). It only means that Proposition 21 alone is not enough to infer the decidability of the $[c]$-free fragment of the modal logics of dynamic mereotopologies. Similar comments can be made about Proposition 22.

Conclusion

Apart from these minor presentation defects, I consider that the thesis of Vladislav Nenchev contains a lot of interesting and new results in mathematics (Mathematical Logic) and that their proofs — in spite of their high complexity and unusual length — have been rigourously presented in a readable form. For all the above-mentioned reasons, I consider that Vladislav Nenchev deserves to receive the title of doctor in mathematics (Mathematical Logic) of Sofia University.